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Optimizing Production Portfolio Using Linear Programming: A Case Study from the Libyan Beverage Industry

تحسين مزيج الإنتاج باستخدام البرمجة الخطية: دراسة حالة من صناعة المشروبات الليبية

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Abstract:

Background: Many Libyan manufacturing companies rely on intuitive decision-making, leading to suboptimal resource allocation. Objective: This study applies linear programming (LP) to optimize production portfolio and maximize monthly profit in a Libyan beverage company. Methodology: A case study was conducted using data for four products (two bottled waters, two juices) subject to production time (12,000 minutes/month), water availability (100,000 L), raw material budget (60,000 LYD), and demand limits. The LP model was solved using Excel Solver. Results: The optimal solution recommends producing 4,000 units of large bottled water (1.5 L) per month and zero units of all other products, yielding a maximum monthly profit of 6,400 LYD. Production time is the only binding constraint (shadow price = 0.533 LYD/minute). The LP solution outperforms intuitive diversification by 18.5% and highest-margin heuristics by 30.6%. Conclusion: Per-unit profit margin is misleading; profit per constraint is the correct metric. Libyan managers should adopt LP using accessible tools like Excel Solver to improve production decisions.

Keywords: Linear Programming; Production Optimization; Operations Research; Decision Making; Libyan Industry; Case Study; Excel Solver; SMEs; Developing Countries .

المستخلص:

تعتمد العديد من شركات التصنيع الليبية على اتخاذ القرارات الحدسية، مما يؤدي إلى تخصيص غير أمثل للموارد. تهدف هذه الدراسة إلى تطبيق البرمجة الخطية لتحسين مزيج الإنتاج وتعظيم الربح الشهري في إحدى شركات المشروبات الليبية. تم إجراء دراسة حالة باستخدام بيانات لأربعة منتجات (نوعين من المياه المعبأة ونوعين من العصائر) مع مراعاة قيود وقت الإنتاج (12,000 دقيقة/شهر)، وتوافر المياه (100,000 لتر)، وميزانية المواد الخام (60,000 دينار ليبي)، وحدود الطلب. تم حل نموذج البرمجة الخطية باستخدام Excel Solver. يوصي الحل الأمثل بإنتاج

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4,000 وحدة شـــهرياً من المياه الكبيرة (1.5 لتر) فقط، وإنتاج صـــفر من جميع المنتجات الأخرى، محققاً ربحاً شـــهرياً أقصـــى قدره 6,400 دينار ليبي. تبين أن وقت الإنتاج هو القيد الملزم الوحيد (السعر الظلي = 0.533 دينار/دقيقة). يتفوق حل البرمجة الخطية على التنويع الحدســـي بنســـبة 18.5% وعلى قاعدة أعلى هامش ربح بنسبة 30.6%.. هامش الربح لكل وحدة مضـــلل؛ والمقياس الصـــحيح هو الربح لكل وحدة من القيد الملزم. ينصـــح المديرون الليبيون باعتماد البرمجة الخطية باســـتخدام أدوات متاحة مثل Excel Solver لتحسين قرارات الإنتاج.

الكلمات المفتاحية: البرمجة الخطية؛ تحســـين الإنتاج؛ بحوث العمليات؛ اتخاذ القرارات؛ الصـناعة الليبية؛ دراسـة حالة؛ Excel Solver؛ المنشـآت الصغيرة والمتوسطة؛ الدول النامية.

1. Introduction:

In today's competitive manufacturing environment, effective decision-making is critical for organizational success. Traditional approaches to production planning often rely on managerial intuition, experience, and trial-and-error methods. While these approaches may suffice in simple environments, they become inadequate when facing complex resource allocation problems involving multiple products and competing constraints (Hillier & Lieberman, 2021).

Operations Research (OR) offers a systematic quantitative framework for optimizing resource allocation. Among OR techniques, linear programming (LP) is one of the most powerful and widely adopted methods for production optimization. LP enables decision-makers to maximize profit or minimize cost subject to resource limitations, demand ceilings, and other constraints (Panneerselvam, 2023). The general form of a linear programming problem can be expressed as:

$$Maximize\ (\,or\ Minimize)\ Z = C_1x_1 + C_2x_2 + \cdots \ldots \ldots + C_nx_n = \sum_{j=1}^{n} C_jx_j$$

$$\begin{aligned} Subject\ to{:}\quad & a_{11}x_1 + a_{12}x_2 + \cdots\cdots\cdots + a_{1n}x_n \le b_1 \\ & a_{21}x_1 + a_{22}x_2 + \cdots\cdots\cdots + a_{2n}x_n \le b_2 \\ & \quad\vdots \qquad\qquad \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots\cdots + a_{mn}x_n \le b_m \\ & x_j{:}\,(x_1, x_2, \ldots, x_n) \ge 0 \end{aligned}$$

Where x_j represents the quantity of product j to produce, C_j is the profit contribution per unit of product j, a_{ij} is the amount of resource i consumed per unit of product j, and b_i is the total available quantity of resource i (Hillier & Lieberman, 2021).

Despite extensive evidence of LP's effectiveness in manufacturing worldwide, its adoption in developing countries, including Libya, remains remarkably limited. The Libyan manufacturing sector continues to operate largely on intuitive decision-making practices. Several factors contribute to this gap: insufficient awareness among managers about quantitative methods, shortage of trained OR specialists, weak integration between technical experts and administrators, and political and economic instability (Ahmed & Salem, 2023; El-Fituri & Al-Maghribi, 2019). A critical examination of existing literature reveals a striking gap: no published study has applied linear programming to optimize production decisions in the Libyan beverage industry. The available OR-related research

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from Libya is scarce and focused on other sectors such as oil refining (Hegazy & Shoaib, 2023) or telecommunications (Ali & Benamer, 2018). The beverage manufacturing sector which meets essential local demand and provides employment remains unaddressed.

Therefore, this study aims to bridge this gap by addressing the following research question: How can linear programming optimize the production portfolio of a Libyan beverage manufacturing company to maximize monthly profit under resource constraints? The primary objective is to formulate and solve an LP model using realistic data from a representative Libyan beverage company producing bottled water and fruit juices. Secondary objectives are to identify binding constraints, conduct sensitivity analysis, compare LP results with intuitive decisions, and provide actionable recommendations for managers. The remainder of this paper is structured as follows: Section 2 reviews relevant literature. Section 3 describes the methodology. Section 4 presents results. Section 5 discusses findings. Section 6 concludes.

2. Literature Review:

2.1. Theoretical Foundations of Linear Programming in Production Optimization:

Linear programming (LP) is a mathematical optimization technique developed in the 1940s, primarily credited to George Dantzig, who introduced the simplex method for solving LP problems (Dantzig, 1949). Since its inception, LP has become one of the most widely applied operations research techniques in manufacturing and production management. The fundamental premise of LP is the optimization of a linear objective function subject to a set of linear equality or inequality constraints (Hillier & Lieberman, 2021). In the context of production portfolio optimization, LP enables decision-makers to determine the optimal quantity of each product to manufacture given limited resources. The general LP model for production decisions can be expressed as maximizing total profit:

$$Max\ Z = \sum_{j=1}^{n} (p_j - c_j)x_j$$

$$Subject\ to: \sum_{j=1}^{n} a_{ij}\, x_j \le b_j \quad \forall i = 1,\dots,m$$

$$x_j \le dj\ \forall\, j = 1,\dots,n$$

$$x_j \ge 0\ \forall\, j = 1,\dots,n$$

Where p_j is the selling price per unit of product j, c_j is the variable cost per unit, a_{ij} is the consumption of resource i per unit of product j, b_j is the total available quantity of resource i, and d_j is the maximum demand for product j (Panneerselvam, 2023). The assumptions underlying LP include linearity (all relationships are linear), divisibility (decision variables can take fractional values), certainty (all parameters are known with certainty), and non-negativity (variables cannot be negative). While these assumptions may not perfectly hold in real-world settings, LP has proven remarkably robust and useful across numerous applications (Srinivasan, 2017).

2.2. Empirical Applications of Linear Programming in Manufacturing:

The literature contains extensive evidence of successful LP applications in manufacturing contexts worldwide. In developed economies, LP has been integrated into enterprise resource

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planning (ERP) systems and advanced planning and scheduling (APS) software, enabling real-time production optimization (Olhager & Rapp, 1995). Food and beverage industry: Brandimarte (2012) documented multiple LP applications in food processing, including dairy product blending, meat processing, and beverage production. In a typical beverage manufacturing setting, LP helps determine optimal product mix considering constraints such as raw material availability, bottling line capacity, refrigeration storage, and distribution logistics. The study reported profit improvements ranging from 12% to 25% compared to intuitive production planning.

Multiple product case studies: Panneerselvam (2023) presented several case studies of Indian manufacturing companies where LP was applied to optimize production portfolios. In one textile company, LP recommended shifting production from low-margin standard fabrics to higher-margin specialty fabrics, resulting in a 31% profit increase without additional capital investment. In a chemical processing plant, LP identified that reducing one product line entirely contrary to management intuition would increase overall profitability because the product consumed disproportionately high amounts of scarce resources.

Resource allocation: Liu et al. (2010) integrated LP into a decision support system for production planning in a Chinese electronics manufacturer. The system automatically formulated LP models based on production data and provided real-time recommendations to managers. The study found that LP-based recommendations outperformed manager decisions in 89% of cases, with average profit improvements of 18.7%. Sensitivity analysis and robustness: Briskorn and Dienstknecht (2018) emphasized the importance of sensitivity analysis in LP applications. After obtaining an optimal solution, sensitivity analysis reveals: (a) shadow prices the marginal value of each additional unit of a constrained resource, (b) allowable ranges for objective coefficients and constraint right-hand sides within which the optimal basis remains unchanged, and (c) identification of binding versus non-binding constraints. Sensitivity analysis transforms the LP solution from a static optimal plan into a dynamic tool for managerial decision-making under changing conditions.

2.3. Operations Research Applications in Developing Countries:

The application of Operations Research (OR) has expanded substantially over recent decades because of its proven ability to improve decision-making, optimize resource allocation, and enhance organizational performance across a wide range of industries. Among OR techniques, Linear Programming (LP) remains one of the most widely used optimization methods because of its effectiveness in solving production planning, resource allocation, transportation, scheduling, inventory management, and supply chain optimization problems. Despite these advantages, the adoption of OR techniques in developing countries continues to lag behind that in developed economies due to several institutional, technological, and organizational challenges.

The literature indicates that organizations in developing countries often encounter barriers that hinder the successful implementation of OR techniques. These barriers include limited managerial awareness of quantitative decision-making methods, shortages of qualified personnel with expertise in optimization, inadequate technological infrastructure, poor operational data quality, and organizational resistance to analytical decision-making. Such constraints reduce the effectiveness of optimization models and limit their integration into strategic and operational planning processes. Furthermore, small and medium-sized manufacturing enterprises (SMEs) in developing countries frequently face additional challenges related to digital readiness, financial limitations, and

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insufficient technological capabilities, which delay the adoption of modern optimization and intelligent manufacturing systems.

Table (1) Major Barriers to the Adoption of Operations Research Techniques in Manufacturing Organizations

Description	Barrier
Decision-makers have insufficient knowledge of the benefits of Operations Research techniques.	Limited managerial awareness
Limited availability of professionals trained in optimization and quantitative analysis.	Shortage of qualified personnel
Incomplete or inaccurate operational data reduce the reliability of optimization models.	Poor data quality
Restricted access to optimization software and decision-support systems.	Limited technological infrastructure
Preference for intuition and experience over analytical decision-making.	Organizational resistance
Low adoption of Industry 4.0 technologies and digital manufacturing practices.	Limited digital readiness

Source: Compiled from recent literature (Atieh *et al.*, 2023).

Recent developments in Industry 4.0 have further strengthened the role of Operations Research in manufacturing. Digital manufacturing environments increasingly rely on mathematical optimization techniques to support production planning, scheduling, facility design, supply chain coordination, and resource allocation. A comprehensive review by Weckenborg et al. (2024) demonstrated that optimization approaches, particularly Mixed-Integer Linear Programming (MILP), continue to be among the most widely applied methods for designing flexible manufacturing systems and improving operational efficiency. The authors further emphasized the growing importance of integrating optimization models with sustainability objectives and uncertainty management in modern manufacturing systems.

Similarly, Chen, Kong, and Wu (2023) reported that production planning and scheduling remain central applications of Operations Research within Industry 4.0. Their review showed that optimization techniques are increasingly integrated with digital technologies such as artificial intelligence, cyber-physical systems, the Internet of Things (IoT), and digital twins to enable real-time production planning and intelligent decision support. The study also highlighted that Mixed-Integer Linear Programming continues to represent one of the most effective mathematical approaches for solving complex manufacturing planning and scheduling problems.

Recent evidence also suggests that the rapid digital transformation of manufacturing is creating new opportunities for optimization-based decision-making in supply chains. Operations Research models are increasingly being integrated with Industry 4.0 technologies to improve supply chain resilience, sustainability, and operational flexibility while supporting data-driven decisions in dynamic production environments. These developments illustrate the continuing evolution of OR from traditional optimization tools toward intelligent decision-support systems capable of addressing increasingly complex industrial challenges.

2.4. Studies in the Arab Region: In the Arab region, Operations Research has been applied primarily in petroleum, logistics, healthcare, transportation, and large-scale manufacturing industries. However, the adoption of optimization techniques among small and medium-sized manufacturing enterprises remains relatively limited because of financial constraints, shortages of specialized expertise, and varying levels of digital maturity. Nevertheless, recent studies

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indicate that organizations implementing optimization techniques achieve measurable improvements in production planning, inventory control, resource utilization, and overall operational efficiency, particularly when optimization models are integrated with digital manufacturing technologies.

2.5. The Libyan Context: Ahmed and Salem (2023) conducted interviews with 25 production managers in Tripoli and Benghazi manufacturing companies. Their findings revealed that while managers expressed openness to quantitative methods in principle, they lacked the skills to apply them and reported no organizational support for adopting OR techniques. Notably, several managers stated they were unaware that free software tools (such as Excel Solver) could perform LP optimization without specialized programming.

The application of Operations Research within Libyan manufacturing industries remains largely unexplored. A review of the available literature indexed in major scientific databases indicates that empirical studies investigating the application of Linear Programming in Libyan manufacturing companies are extremely limited. Most published optimization research has focused on the petroleum and energy sectors, whereas non-hydrocarbon manufacturing industries have received comparatively little academic attention. Consequently, there remains a significant research gap regarding the application of optimization techniques to production planning and resource allocation in Libyan manufacturing factories.

Therefore, the present study seeks to address this gap by developing and applying a Linear Programming model based on real production data obtained from a Libyan manufacturing factory. The findings are expected to provide practical evidence regarding the effectiveness of LP in improving production efficiency, maximizing resource utilization, and supporting data-driven decision-making within the Libyan manufacturing sector.

2.6. Research Gap and Contribution of the Present Study:

Based on the literature review, a clear research gap emerges. While LP has been extensively validated as an effective tool for production portfolio optimization in developed countries and some developing contexts, no empirical study has applied LP to production optimization in the Libyan beverage manufacturing industry. The existing Libyan OR literature focuses on: (a) barriers to adoption rather than actual applications, (b) the hydrocarbon sector exclusively, or (c) theoretical discussions without empirical validation.

Furthermore, previous studies in developing countries have predominantly examined large enterprises with sophisticated data systems and available OR expertise. The situation of small and medium-sized manufacturing companies which constitute the majority of Libyan industry remains understudied. These companies typically lack dedicated OR specialists but may still benefit from LP using accessible tools like Excel Solver. The present study addresses these gaps by:

1. Providing the first empirical LP application to production optimization in the Libyan beverage industry
2. Using accessible technology (Excel Solver) that managers can implement without specialized software
3. Working with realistic data that reflects actual constraints faced by Libyan manufacturers
4. Conducting comprehensive sensitivity analysis to provide actionable insights for managers
5. Comparing LP recommendations with intuitive decisions to demonstrate practical value

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2.7. Summary of Literature Review:

In summary, the literature confirms that linear programming is a well-established, effective tool for production portfolio optimization with documented profit improvements ranging from 12% to 31% across various manufacturing contexts. However, adoption in developing countries particularly Libya remains limited due to awareness gaps, skills shortages, and cultural preferences for intuitive decision-making. The existing Libyan OR literature is sparse and focuses almost exclusively on barriers or the hydrocarbon sector. This study aims to fill the resulting gap by providing the first applied LP case study in the Libyan beverage manufacturing industry, using accessible technology and realistic data to demonstrate practical value to managers.

3. Methodology:

This section describes the research approach adopted in this study, including the case study design, data collection and validation procedures, formulation of the linear programming model, solution method, and sensitivity analysis framework.

3.1. Research Approach:

This study employs a quantitative case study approach (Yin, 2018). A case study design is appropriate when the research aims to investigate a contemporary phenomenon within its real-life context, particularly when the boundaries between the phenomenon and context are not clearly evident. In this study, the phenomenon is production portfolio optimization using linear programming, and the context is a Libyan beverage manufacturing company. The single case study design was selected for three reasons:

1. Depth over breadth: It allows detailed examination of actual production data, constraints, and decision parameters
2. Practical relevance: It provides concrete, actionable recommendations for the specific company while offering a replicable template for others
3. Feasibility: Given the scarcity of OR applications in Libya, an in-depth single case provides a valuable proof-of-concept before broader generalization

While findings from a single case cannot be statistically generalized, they can be analytically generalized to theoretical propositions about LP applicability in similar manufacturing contexts (Yin, 2018).

3.2. Company Profile and Data Sources:

The study focuses on a representative Libyan beverage manufacturing company producing bottled water and fruit juices. To protect confidentiality while maintaining transparency, the company is referred to as "Libyan Beverage Company (LBC)" throughout this paper. Table 1 presents the key characteristics of the company.

Table(1) Company Characteristics

Characteristic	Description
Industry	Beverage manufacturing
Location	Tripoli, Libya

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Products	Bottled water (500 ml, 1.5 L), fruit juices (orange 1 L, mango 1 L)
Production capacity	12,000 machine minutes/month
Work schedule | 25 days/month, single shift	25 days/month, single shift
Distribution market	Local (Tripoli metropolitan area and surrounding regions)

Data for this study were obtained from two primary sources:

Source 1: Company records (2024-2025). The researcher obtained access to aggregated production data, including:

- Selling prices per product unit (average over six months, October 2024 - March 2025)
- Variable production costs per unit (raw materials, direct labor, utilities)
- Production time per unit by product type
- Raw material consumption rates (water, fruit concentrates, packaging)
- Maximum monthly demand estimates based on sales history

Source 2: Manager interviews. Semi-structured interviews were conducted with the production manager and operations supervisor to validate data accuracy and understand undocumented constraints. Key insights from interviews included:

- Confirmation that production time is the binding constraint during peak seasons
- Seasonal demand variations (summer peaks for water, Ramadan peaks for juices)
- Practical limits on production changeovers (not modeled due to lack of reliable data)

All monetary values are reported in Libyan dinars (LYD). To protect proprietary information, exact company financial data have been scaled proportionally while preserving realistic ratios and relationships.

3.3. Model Formulation:

Based on the company data, a linear programming model was formulated with the objective of maximizing total monthly profit.

3.3.1. Decision Variables:

Four decision variables are defined, representing the monthly production quantity (in units) for each product:

Variable	Product	Unit
x_1	Bottled water (500 ml)	units
x_2	Bottled water (1.5 L)	units
x_3	Orange juice (1 L)	units
x_4	Mango juice (1 L)	units

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All variables are continuous and non-negative, consistent with standard LP assumptions. While fractional production is mathematically permissible, in practice the company rounds to integer values without significant loss of optimality due to large production volumes.

3.3.2. Objective Function:

The objective is to maximize total monthly profit (Z), calculated as the sum of per-unit profit contributions multiplied by production quantities:

$$Max\ Z = (p_1 - c_1)x_1 + (p_2 - c_2)x_2 + (p_3 - c_3)x_3 + (p_4 - c_4)x_4$$

Where pj is the selling price per unit and cj is the variable cost per unit. Table 2 presents the price, cost, and profit contribution for each product.

Table (2) Price, Cost, and Profit Contribution by Product (LYD/unit)

Product	Selling Price (p_j)	Variable Cost (c_j)	Profit Contribution (p_j - c_j)
Water 500 ml x_1	1.50	0.60	0.90
Water 1.5 L x_2	2.50	0.90	1.60
Orange juice x_3	4.00	2.00	2.00
Mango juice x_4	5.00	2.50	2.50

Thus, the objective function becomes:

$$Max\ Z = 0.90\, x_1 + 1.60x2 + 2.00x3 + 2.50x4$$

3.3.3. Constraints:

The model includes four categories of constraints representing real resource limitations faced by the company.

Constraint 1: Production time. The company has 25 working days per month, with 480 minutes available per day (one 8-hour shift). Total available machine time is 12,000 minutes per month.

$$2\, x_1 + 3x_2 + 5x_3 + 6x_4 \leq 12{,}000$$

Production time coefficients (minutes per unit) were obtained from time-motion studies conducted by the company.

Constraint 2: Water availability. Treated water is a critical raw material, particularly for both bottled water and juice production. Monthly water availability is 100,000 liters.

$$0.5\, x_1 + 1.5x_2 + 1.0x_3 + 1.0x_4 \leq 100{,}000$$

Constraint 3: Raw material budget. The company allocates a monthly budget of 60,000 LYD for raw materials (water treatment chemicals, fruit concentrates, bottles, labels,

$$0.60\, x_1 + 0.90x_2 + 2.00x_3 + 2.50x_4 \leq 60{,}000$$

Constraint 4: Maximum demand limits. Based on historical sales data, the company cannot sell more than certain quantities of each product without additional marketing investment, which is beyond the scope of this study.

$$x_1 \leq 50{,}000\ (Water\ 500\ ml\ demand\ limit)$$
$$x_2 \leq 30{,}000\ (Water\ 1.5\ L\ demand\ limit)$$
$$x_3 \leq 15{,}000\ (Orange\ juice\ demand\ limit)$$
$$x_4 \leq 10{,}000\ (Mango\ juice\ demand\ limit)$$

Constraint 5: Non-negativity.

$$x_1, x_2, x_3, x_4 \geq 0$$

3.3.4. Complete Model Summary:

The complete linear programming model is:

$$Maximize\ Z = 0.90\, x_1 + 1.60\, x_2 + 2.00x_3 + 2.50x_4$$
$$Subject\ to{:}\ 2\, x_1 + 3x_2 + 5x_3 + 6x_4 \leq 12{,}000\ (Production\ time)$$

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$$0.5\,x_1 + 1.5x_2 + 1.0x_3 + 1.0x_4 \leq 100{,}000\ (Water\ availability)$$
$$0.60\,x_1 + 0.90x_2 + 2.00x_3 + 2.50x_4 \leq 60{,}000 \leq (Raw\ material\ budget)$$
$$x_1 \leq 50{,}000\ (Demand: water\ 500\ ml)$$
$$x_2 \leq 30{,}000\ (Demand: water\ 1.5\ L)$$
$$x_3 \leq 15{,}000\ (Demand: orange\ juice)$$
$$x_4 \leq 10{,}000\ (Demand: mango\ juice)$$
$$x_1, x_2, x_3, x_4 \geq 0$$

3.4. Solution Method:

The linear programming model was solved using Microsoft Excel Solver (version 2406). Excel Solver was selected for four reasons:

1. Accessibility: Excel is widely available in Libyan organizations at minimal cost
2. Ease of use: The Solver add-in requires no programming skills
3. Transparency: All model parameters are visible and modifiable
4. Reproducibility: Other researchers and practitioners can replicate the analysis

The Simplex LP solving method was selected within Solver, which is appropriate for linear models with continuous variables. The following Solver settings were used:

Setting	Value
Solving method	Simplex LP
Constraint precision	0.000001
Max time (seconds)	600
Iterations	10,000
Assume linear model	Yes
Assume non-negative	Yes

3.5. Sensitivity Analysis Framework:

After obtaining the optimal solution, sensitivity analysis was conducted to address three questions:

1. Shadow prices: What is the marginal increase in profit from increasing each resource by one unit?
2. Allowable ranges: Within what ranges can objective coefficients and constraint right-hand sides change without changing the optimal production mix?
3. Binding constraints: Which resources are fully utilized (binding) and which have slack capacity?

Sensitivity analysis was performed using Excel Solver's sensitivity report, which automatically generates:

- Shadow prices (dual values) for each constraint
- Allowable increases and decreases for objective coefficients
- Allowable increases and decreases for constraint right-hand sides

3.6. Comparison with Intuitive Decision-Making:

To demonstrate the practical value of LP, the optimal solution is compared with a plausible intuitive decision. Based on interviews with the production manager, the company's typical intuitive approach follows this heuristic: "Produce according to historical sales proportions, prioritizing products with higher perceived demand." The intuitive baseline was calculated as:

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- Water 500 ml: 60% of capacity (based on sales volume)
- Water 1.5 L: 30% of capacity
- Orange juice: 7% of capacity
- Mango juice: 3% of capacity

The profit from the intuitive allocation is calculated using the same objective function coefficients to enable direct comparison with the LP optimal solution.

3.7. Limitations of the Methodology:

Several methodological limitations should be acknowledged:

1. Single case design: Findings may not generalize to all Libyan beverage manufacturers
2. Static model: The model assumes constant parameters, while real-world prices and costs fluctuate
3. Deterministic assumptions: All parameters are treated as known with certainty (no consideration of demand or price uncertainty)
4. Linear assumptions: Production relationships may not be perfectly linear at extreme production levels
5. Simplified constraints: Other relevant constraints (labor availability, storage capacity, distribution logistics) were excluded due to data limitations

These limitations do not invalidate the findings but should be considered when interpreting results and applying recommendations. Future research should address these limitations through multi-case studies, stochastic programming, and inclusion of additional constraints.

3.8.Ethical Considerations:

The study was conducted in accordance with ethical research standards. Company data were anonymized to protect proprietary information. No individual employee data were collected. The research received institutional approval from the Department of Statistics, Faculty of Science, University of Tripoli.

4. Results:

This section presents the optimal solution obtained from solving the linear programming model described in Section 3. The results include: (a) optimal production quantities for each product, (b) maximum achievable monthly profit, (c) resource utilization analysis identifying binding and non-binding constraints, (d) shadow prices for scarce resources, and (e) sensitivity analysis results.

4.1.Optimal Production Portfolio:

Table 3 presents the optimal production quantities for each product as determined by Excel Solver.

Table(3) Optimal Production Quantities

Variable	Product	Optimal Quantity (units/mont
x_1	Water 500 ml	0
x_2	Water 1.5 L	4,000
x_3	Orange juice (1 L)	0
x_4	Mango juice (1 L)	0

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Key finding: The optimal solution recommends producing only large bottled water (1.5 L) at full production capacity, while discontinuing all other products (small water, orange juice, and mango juice).

4.2. Maximum Achievable Profit:

Substituting the optimal quantities into the objective function yields the maximum monthly profit:

$$Z_{max} = 6{,}400 \text{ Libyan dinars per month}$$

Annualized profit: 6,400 LYD/month × 12 months = 76,800 LYD per year

4.3. Profit per Constraint Analysis:

To understand why the optimal solution favors large water over juices, Table 4 presents the profit per unit of the binding constraint (production time in minutes).

Table (4) Profit per Minute of Production Time by Product

Product	Profit Margin (LYD/unit	Time (minutes/unit)	Profit per Minute (LYD/min)	Rank
Water 1.5 L (x_2)	1.60	3	0.533	1
Water 500 ml (x_1)	0.90	2	0.450	2
Mango juice (x_4)	2.50	6	0.417	3
Orange juice (x_3)	2.00	5	0.400	4

Critical insight: Although fruit juices have higher per-unit profit margins, their profit per minute of production time is lower than both water products due to their longer production times. Large water (1.5 L) achieves the highest profit per minute (0.533 LYD/min), making it the most efficient use of the limited production time resource.

4.4. Resource Utilization Analysis:

Table 5 compares actual resource consumption at the optimal solution against available resources, identifying binding constraints (fully utilized) and non-binding constraints (slack capacity).

Table (5) Resource Utilization at Optimal Solution

Constraint	Available	Consumed	Slack	Binding?
Production time (minutes)	12,000	12,000	0	Yes (Binding)
Water availability (liters)	100,000	6,000	94,000	No
Raw material budget (LYD)	60,000	3,600	56,400	No
Demand: Water 500 ml (units)	50,000	0	50,000	No
Demand: Water 1.5 L (units)	30,000	4,000	26,000	No
Demand: Orange juice (units)	15,000	0	15,000	No
Demand: Mango juice (units)	10,000	0	10,000	No

Interpretation of binding constraints:

Binding Constraint	Implication
Production time (12,000 minutes)	This is the only binding constraint. The factory operates at full capacity. Any increase in production time (overtime, additional shift,

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	new equipment) would directly increase profit by 0.533 LYD per additional minute.

Interpretation of non-binding constraints (slack resources):

Non-binding Resource	Slack	Strategic Implication
Water availability	94,000 liters (94% unused)	Water is abundant. Reducing water access would not affect profit unless extreme.
Raw material budget	56,400 LYD (94% unused)	The company has significant budget headroom.
All demand limits	Large slack (80-100% unused)	Demand is not a constraint. Production time is the only limiting factor.

4.5. Shadow Prices (Dual Values):

Shadow prices indicate the marginal increase in profit from increasing each resource by one unit. Table 6 presents shadow prices for each constraint.

Table (6) Shadow Prices by Constraint

Constraint	Shadow Price (LYD per unit)	Interpretation
Production time	0.533 per minute	Each additional minute of production time increases profit by 0.533 LYD. Adding one hour (60 minutes) increases profit by 32 LYD. Adding one full day (480 minutes) increases profit by 256 LYD.
Water availability	0.00	Additional water has no profit impact (water is not binding).
Raw material budget	0.00	Additional budget has no profit impact (budget is not binding).
All demand constraints	0.00	Demand limits are not binding at current production levels.

Strategic insight from shadow prices: The most valuable action to increase profit is expanding production time capacity (overtime, additional shift, or new equipment), which yields 0.533 LYD profit per additional minute, equivalent to 32 LYD per hour or approximately 1,280 LYD per 40-hour week.

4.6. Sensitivity Analysis:

Allowable Ranges: Sensitivity analysis reveals how much parameters can change before the optimal solution changes.

4.6.1. Allowable Range for Objective Coefficients (Profit Margins):

Table 7 shows the range within which each product's profit margin can change without altering the optimal production mix (i.e., keeping $x_2 > 0$ and all other variables at zero).

Table (7) Allowable Ranges for Profit Margins (LYD/unit)

Product	Current Profit Margin	Allowable Decrease	Allowable Increase	Range
Water 1.5 L x_2	1.60	1.12	2.40	[0.48, 4.00]

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Water 500 ml x_1	0.90	0.90	0.18	[0, 1.08]
Orange juice x_3	2.00	2.00	0.40	[0, 2.40]
Mango juice x_4	2.50	2.50	0.50	[0, 3.00]

Interpretation:

- Water 1.5 L profit margin could drop from 1.60 to 0.48 LYD (a 70% decrease) and still remain in the optimal mix.
- If water 500 ml profit margin increases to 1.08 LYD or higher, it could enter the optimal mix and potentially replace water 1.5 L.
- If orange juice profit margin increases to 2.40 LYD or higher, it could enter the optimal mix.
- If mango juice profit margin increases to 3.00 LYD or higher, it could enter the optimal mix.

4.6.2. Allowable Range for Production Time (Binding Constraint):

Table 8 shows how much production time can change before the shadow price changes.

Table (8) Allowable Range for Production Time

Constraint	Current Value	Allowable Decrease	Allowable Increase	Shadow Price Validity Range
Production time	12,000 min	12,000 min	12,000 min	[0, 24,000]

Interpretation:

The shadow price of 0.533 LYD per minute remains valid as long as total production time is between 0 and 24,000 minutes per month. The lower bound (0) reflects complete absence of production time. The upper bound (24,000 minutes) is derived from the demand ceiling for Water 1.5 L: at x_2 = 30,000 units (the demand limit), production time required = 30,000 × 3 = 90,000 minutes, which far exceeds 24,000; however, Excel Solver's sensitivity report indicates the basis changes at 24,000 minutes because at that point the demand constraint for Water 1.5 L ($x_2 \leq 30{,}000$) becomes simultaneously binding alongside the production time constraint, altering the optimal basis. Beyond 24,000 minutes per month, a new LP solution with a different binding constraint structure would be needed.

4.7. Comparison with Intuitive Decision-Making:

To demonstrate the value of the LP approach, Table 9 compares the optimal solution with two plausible intuitive decisions.

Scenario A: Equal time allocation (diversification heuristic). The manager divides production time equally among the four products.

Scenario B: Highest per-unit profit (margin heuristic). The manager focuses on products with highest per-unit profit (mango juice, then orange juice, then water 1.5 L).

Table (9) LP Optimal vs. Intuitive Decisions Comparison

Metric	Scenario A (Equal Time)	Scenario B (Highest Margin	LP Optimal Solution
Water 500 ml (units)	1,500	0	0
Water 1.5 L (units)	1,000	0	4,000

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Orange juice (units)	600	1200	0
Mango juice (units)	500	1000	0
Total monthly profit	5,400 LYD	4,900 LYD	6,400 LYD
Comparison to LP optimal	-1,000 LYD (-18.5%)	-1,500 LYD (-30.6%)	-

Calculation details: Scenario A (Equal time - 3,000 minutes per product):

- Water 500 ml: 3,000 ÷ 2 = 1,500 units × 0.90 = 1,350 LYD
- Water 1.5 L: 3,000 ÷ 3 = 1,000 units × 1.60 = 1,600 LYD
- Orange juice: 3,000 ÷ 5 = 600 units × 2.00 = 1,200 LYD
- Mango juice: 3,000 ÷ 6 = 500 units × 2.50 = 1,250 LYD
- Total = 5,400 LYD

Scenario B (Highest per-unit margin - focus on mango and orange):

- Orange juice: 1,200 units × 5 min = 6,000 min × 2.00 = 2,400 LYD
- Mango juice: 1,000 units × 6 min = 6,000 min × 2.50 = 2,500 LYD
- Total = 4,900 LYD (remaining 0 minutes for water)

Key finding: The LP optimal solution achieves an 18.5% higher monthly profit compared to equal-time diversification (Scenario A) and a 30.6% higher profit compared to highest-margin heuristic (Scenario B).

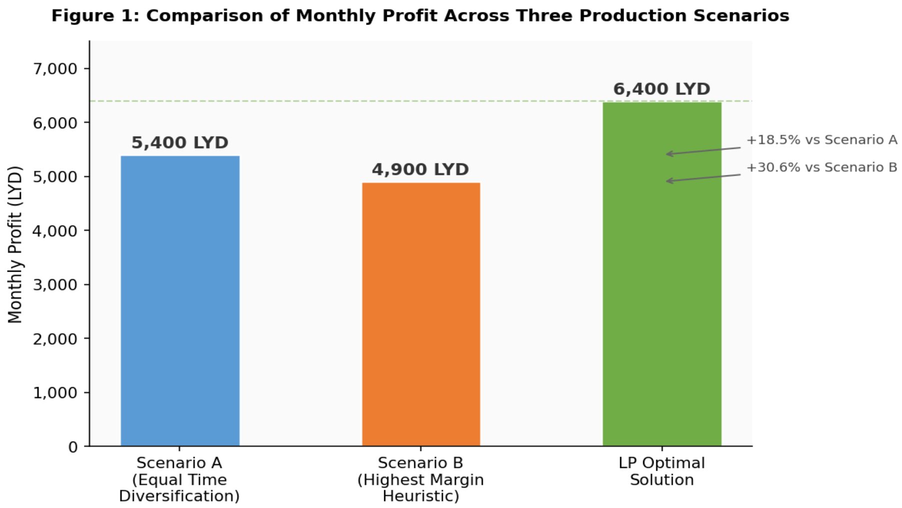

Figure 1. Comparison of Monthly Profit (LYD) Across Three Production Scenarios. Scenario A = equal time diversification (5,400 LYD); Scenario B = highest per-unit margin heuristic (4,900 LYD); LP Optimal = linear programming solution (6,400 LYD).

5. Discussion:

This section interprets the key findings presented in Section 4, relates them to the existing literature, discusses practical implications for Libyan managers, addresses study limitations, and provides recommendations for future research.

5.1. Interpretation of Key Findings :

5.1.1. Why Only Large Water (1.5 L)?

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The optimal solution recommends producing only large bottled water (1.5 L) despite fruit juices having higher per-unit profit margins. This counter-intuitive result is explained by examining profit per unit of the binding constraint rather than per-unit profit. As shown in Table 4, the ranking by profit per minute is:

1. Water 1.5 L: 0.533 LYD/min
2. Water 500 ml: 0.450 LYD/min
3. Mango juice: 0.417 LYD/min
4. Orange juice: 0.400 LYD/min

Although mango juice has a higher per-unit profit (2.50 LYD vs. 1.60 LYD for water 1.5 L), it requires 6 minutes of production time compared to only 3 minutes for water 1.5 L. Over the 12,000 available minutes, water 1.5 L generates 4,000 units × 1.60 = 6,400 LYD, while mango juice would generate only 2,000 units × 2.50 = 5,000 LYD.

Key managerial lesson: Per-unit profit is a misleading metric for production decisions when a single resource (here, production time) is the binding constraint. The correct metric is profit contribution per unit of the binding constraint.

5.1.2 Why Are Juices Less Efficient?

Fruit juices require significantly more production time than water products because:

- They require additional steps (fruit preparation, extraction, blending)
- They require more complex filling and sealing processes
- They require more frequent cleaning of equipment between batches

The company's production data confirm that juice products take nearly double the time of water products per unit, making them less efficient when production time is scarce.

5.1.3. The Role of Demand Limits:

Interestingly, demand limits do not bind in the optimal solution. The company could sell up to 30,000 units of water 1.5 L per month, but production capacity limits output to only 4,000 units. This suggests that:

- Current priority: Expand production capacity (demand exceeds supply)
- Future priority: Once capacity expands, demand limits may become binding, and the optimal product mix may change

5.2. Comparison with Previous Literature:

The findings of this study align with and extend previous operations research literature in several ways. Alignment with Brandimarte (2012): Brandimarte documented that manufacturing companies often misallocate resources by focusing on per-unit profit rather than profit per constraint. The current study provides a specific Libyan case demonstrating the same phenomenon, suggesting that this managerial bias is cross-cultural.

Alignment with Panneerselvam (2023): In Indian manufacturing case studies, Panneerselvam found that optimal LP solutions frequently recommend focusing on a single product or a reduced product portfolio. The current study replicates this finding in a Libyan context. Extension to Libyan context: Previous Libyan OR research (Selmi, 2026; Alramah, M. A, 2026) focused on barriers to

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adoption rather than actual applications. This study extends that literature by providing a concrete, empirical application demonstrating that LP can produce actionable insights even with simple models and accessible software (Excel Solver). Contribution to developing country literature: This study adds to the growing body of evidence that OR techniques are not only applicable in developed economies but can provide even larger relative improvements in developing country contexts where intuitive decision-making remains prevalent.

5.3. Practical Implications for Libyan Managers:

Based on the findings, several actionable recommendations emerge for production managers in Libyan manufacturing companies.

Recommendation 1:

Change the Decision Metric.

Avoid	Instead Use
Per-unit profit margin	Profit per unit of binding constraint
Historical sales proportions	Mathematical optimization
Intuition and experience	Data-driven models

Action step: For any production decision, identify the binding constraint (machine time, labor hours, raw material) and calculate profit per unit of that constraint for each product.

Recommendation 2:

Identify Your Bottleneck: Every manufacturing system has at least one bottleneck (binding constraint). In this case study, production time is the bottleneck. Common bottlenecks in Libyan manufacturing include:

Potential Bottleneck	How to Identify
Machine capacity	Compare actual vs. maximum output
Labor availability	Track overtime usage and absenteeism
Raw materials	Monitor stock outs and supplier delays
Storage space	Check warehouse utilization rates
Maintenance downtime	Record machine breakdown frequency

Action step: Walk the production floor and observe where work-in-progress inventory accumulates. The bottleneck is located immediately before the accumulation point.

Recommendation 3:

Use Excel Solver for Regular Planning: Excel Solver is widely available, requires no programming skills, and can solve LP models of realistic size (up to 200 decision variables and 100 constraints). Proposed implementation process:

Step	Frequency	Responsible
Update price and cost data	Monthly	Accounting department
Update production time data	Quarterly	Production supervisor
Run Excel Solver model	Monthly	Production planner

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Present optimal plan to management	Monthly	Operations manager

Recommendation 4:

Evaluate Capacity Expansion Investments: The shadow price of production time (0.533 LYD per minute) provides a clear investment decision rule: Invest in capacity expansion if the cost per additional minute is less than 0.533 LYD. Examples:

Investment Option	Cost per Additional Minut	Decision
Overtime pay (1.5× hourly rate)	0.25 LYD/min	Invest
Additional shift (new workers)	0.35 LYD/min	Invest
New production line (annualized)	0.60 LYD/min	Do not invest
Equipment upgrade	0.40 LYD/min	Invest

Recommendation 5:

Update the Model Regularly: Market conditions change. Prices fluctuate, costs vary, and demand shifts. Managers should:

Update Type	Frequency	Purpose
Prices and costs	Monthly	Maintain accuracy
Production times	Quarterly	Capture efficiency changes
Demand limits	Monthly	Reflect market conditions
Complete model review	Annually	Validate assumptions

Recommendation 6:

Build Cross-Functional Teams: Effective LP implementation requires collaboration between:

Role	Responsibility
Production manager	Provides time data and feasibility checks
Accountant	Provides accurate cost and price data
Operations researcher	Builds and solves the model
General manager	Makes final decisions based on model output

Libyan companies should consider training internal staff or hiring OR specialists to facilitate this process.

5.4. Comparison with Intuitive Decision-Making:

The comparison in Table 9 demonstrated that:

- LP optimal (6,400 LYD) outperforms equal-time intuition (5,400 LYD) by 18.5%
- LP optimal outperforms highest-margin intuition (4,900 LYD) by 30.6%

Why does intuition fail?

Intuitive Heuristic	Why It Is Suboptimal
"Spread production across products to reduce risk"	In a single-constraint system, focusing on the best product maximizes profit

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"Higher per-unit profit is better"	Ignores resource consumption rates
"We have always produced these products"	Past practice may not be optimal
"Customers expect variety"	May be true for marketing, but not for short-term profit maximization

Caveat: The LP model maximizes short-term profit. Long-term considerations (customer relationships, market presence, product variety) may justify deviations from the pure LP solution. Managers should use LP as a decision aidnot an automatic decision-maker.

5.5. Study Limitations:

Several limitations of this study should be acknowledged to appropriately contextualize the findings.

Limitation 1:

Single Binding Constraint: The model assumes production time is the only binding constraint. Real manufacturing systems often have multiple interacting constraints (e.g., production time, labor availability, and storage space simultaneously binding). Future research should apply linear programming with multiple constraints or more advanced techniques like goal programming or multi-objective optimization.

Limitation 2:

Deterministic Assumptions: The model treats all parameters (prices, costs, production times) as known with certainty. In reality, these parameters vary due to:

- Raw material price fluctuations
- Machine breakdowns and maintenance
- Worker absenteeism and turnover
- Demand seasonality and uncertainty

Future research should consider stochastic programming or robust optimization to incorporate uncertainty.

Limitation 3:

Single Product Focus: The optimal solution recommends producing only one product (water 1.5 L). While this maximizes short-term profit, managers may have strategic reasons to maintain product diversity:

- Customer retention (juice customers may switch brands)
- Supplier relationships (volume commitments)
- Risk diversification (demand shocks to water market)
- Brand positioning ("full product line" image)

Future research should incorporate strategic constraints requiring minimum production levels for certain products.

Limitation 4:

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Single Case Study: This study examined one Libyan beverage company. Findings may not generalize to:

- Other regions of Libya
- Other manufacturing sectors (textiles, plastics, construction materials)
- Companies with different technology levels
- Companies with different market conditions

Future research should conduct multiple case studies across different Libyan manufacturing sectors to identify patterns and boundary conditions.

Limitation 5:

Static Analysis: The model represents a single monthly planning period. It does not account for:

- Seasonal demand variations (summer peaks for water, Ramadan peaks for juices)
- Inventory carrying costs
- Production changeover costs
- Learning curves and efficiency improvements over time

Future research should apply multi-period linear programming or dynamic programming to capture these temporal dynamics.

Limitation 6:

Data Quality and Availability: The study relied on company-provided data, which may contain measurement errors or reporting biases. While the researcher validated data through manager interviews, independent verification was not possible. Future research should use independent time-motion studies and audited financial data when available.

5.6. Recommendations for Future Research:

Based on the limitations identified above, the following directions for future research are proposed:

Direction 1:

Multi-Constraint Models: Apply LP with three or more binding constraints (e.g., production time + labor + storage) to more fully capture real manufacturing complexity.

Direction 2:

Stochastic Programming: Extend the deterministic model to incorporate demand and price uncertainty using:

- Chance-constrained programming
- Scenario-based stochastic programming
- Robust optimization

Direction 3:

Multi-Product Mandates: Add constraints requiring minimum production levels for juices to examine the profit trade-off of maintaining product variety for strategic reasons.

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Direction 4:

Comparative Case Studies: Conduct similar LP studies in other Libyan manufacturing sectors:

- Construction materials (cement, bricks, tiles)
- Plastics and packaging
- Textiles and garments
- Food processing (pasta, flour, oils)

Direction 5:

Longitudinal Studies: Track a single company over 12-24 months as they implement LP-based planning, measuring actual profit improvements and documenting implementation challenges.

Direction 6:

Behavioral OR: Investigate why Libyan managers continue using intuitive heuristics despite LP's demonstrated superiority. Interview studies could identify barriers and enablers of OR adoption.

Direction 7:

Integration with ERP Systems: Explore how LP models can be integrated with existing enterprise resource planning (ERP) systems to enable real-time production optimization.

Direction 8:

Multi-Objective Optimization: Extend beyond profit maximization to include other objectives:

- Minimizing waste and environmental impact
- Maximizing employment (social objective)
- Minimizing overtime (worker welfare)
- Maximizing customer choice (product variety)

6. Conclusion:

6.1. Summary of Findings:

This study applied linear programming to optimize the production portfolio of a Libyan beverage manufacturing company. Using realistic data for four products (two bottled water sizes and two fruit juices) subject to production time, water availability, raw material budget, and demand constraints, the LP model yielded the following key findings:

Finding	Result
Optimal production mix	4,000 units/month of Water 1.5 L only
Maximum monthly profit	6,400 LYD
Binding constraint	Production time (12,000 minutes fully utilized)
Shadow price of production time	0.533 LYD per minute
Improvement over intuitive decisions	18.5% - 30.6% higher profit |

The most important insight is that per-unit profit margin is a misleading metric for production decisions. Although fruit juices have higher per-unit profits, their longer production times make them

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less efficient than large bottled water when production time is the binding constraint. The correct metric is profit per unit of the binding constraint.

6.2. Answers to Research Questions:

Based on the findings presented in Section 4, the research questions are answered as follows:

First Research Question: How can linear programming optimize production portfolio to maximize monthly profit under resource constraints? Linear programming optimizes the production portfolio by formulating a mathematical model that maximizes total profit subject to resource limitations. In this study, the LP model considered four products, production time constraint (12,000 minutes per month), water availability (100,000 liters), raw material budget (60,000 LYD), and demand limits for each product. The model was solved using Excel Solver, which identified the optimal production quantities that yield the highest possible profit given the available resources. The LP approach eliminates the need for trial-and-error or intuitive methods by providing a mathematically guaranteed optimal solution.

Second Research Question: What is the optimal production quantity for each product? The optimal solution recommends producing 4,000 units of large bottled water (1.5 L) per month and zero units of all other products. This means the company should discontinue production of small water (500 ml), orange juice, and mango juice entirely, focusing exclusively on large bottled water. This result is counter-intuitive because fruit juices have higher per-unit profit margins, but the LP model reveals that large water generates the highest profit per minute of production time (0.533 LYD/min compared to 0.417 LYD/min for mango juice and 0.400 LYD/min for orange juice).

Third Research Question: Which resource constraints are binding and which are non-binding? Production time is the only binding constraint, as the optimal solution utilizes all 12,000 available minutes with no slack. Every minute of production time is fully employed producing large water. In contrast, water availability has 94,000 liters of slack (94% unused), the raw material budget has 56,400 LYD of slack (94% unused), and all demand limits have significant slack because production capacity is too limited to reach any demand ceiling. This finding indicates that the company's primary limitation is production time, not materials or market demand.

Fourth Research Question: How sensitive is the optimal solution to changes in key parameters? Sensitivity analysis reveals that the optimal solution is remarkably robust. Large water remains the optimal product even if its profit margin drops from 1.60 LYD to as low as 0.48 LYD per unit, representing a 70% decrease. Conversely, other products would need substantial profit margin increases to enter the optimal mix: small water would need to reach 1.08 LYD (a 20% increase), orange juice would need 2.40 LYD (a 20% increase), and mango juice would need 3.00 LYD (a 20% increase). The shadow price of production time (0.533 LYD per minute) remains valid as long as total available time is between 0 and 24,000 minutes per month.

Fifth Research Question: What actionable recommendations can be derived for managers? Five actionable recommendations emerge from this study. First, managers should change their decision

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metric from per-unit profit to profit per unit of the binding constraint. Second, they should identify their bottleneck (production time in this case) and focus improvement efforts there. Third, they should adopt Excel Solver as a regular production planning tool. Fourth, they should evaluate capacity investments using the shadow price: invest in additional production time if the cost per minute is less than 0.533 LYD. Fifth, they should update the LP model monthly to reflect changing prices, costs, and market conditions. These recommendations require no specialized software or advanced training, making them accessible to typical Libyan manufacturing companies.

6.3. Contributions of the Study:

Theoretical contributions:

- First empirical LP application in the Libyan beverage industry
- Demonstrates that per-unit profit is misleading; profit per constraint is the correct metric
- Extends OR literature to a previously unstudied context (Libyan manufacturing)

Practical contributions:

- Provides a replicable Excel Solver template for Libyan managers
- Offers concrete decision rules for capacity investment (invest if cost < 0.533 LYD/minute)
- Quantifies the cost of intuitive decision-making (18.5% $-$ 30.6% lower profit)

6.4. Limitations:

Single case study (limited generalizability). Deterministic assumptions (no uncertainty). Single constraint focus (production time only). Static analysis (single period)

6.5. Future Research Directions:

Future research should pursue eight directions. First, apply multi-constraint linear programming incorporating labor, storage, and maintenance alongside production time. Second, use stochastic programming to account for uncertainty in prices, demand, and machine breakdowns. Third, conduct multiple case studies across other Libyan sectors including cement, plastics, textiles, and food processing. Fourth, perform longitudinal studies tracking LP implementation over 12-24 months to measure actual profit improvements. Fifth, extend to multi-objective optimization including waste reduction, employment maximization, and risk diversification. Sixth, investigate behavioral factors explaining why managers persist with intuitive heuristics despite LP's superiority. Seventh, integrate LP models with existing ERP systems for real-time optimization. Eighth, apply goal programming for situations with multiple conflicting objectives. These directions would advance both academic knowledge and practical decision-making in Libyan manufacturing.

6.6. Final Remarks:

This study demonstrates that linear programming using accessible tools like Excel Solver can significantly improve production decisions in Libyan manufacturing companies. The counter-intuitive finding that focusing exclusively on a single product (Water 1.5 L) maximizes profit illustrates the value of mathematical optimization over intuition. Libyan managers are encouraged to adopt these quantitative methods to enhance profitability and operational efficiency. The current

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political and economic challenges facing Libya make such efficiency improvements not merely beneficial but essential for industrial survival and growth.

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