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Results On Sb-Bipolar Interval Neutrosophic Set

نتائج حول دالة Sb- Neutrosophic ذات الفترة ثنائية القطب.

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Abstract:

The neutrosophic set is mathematical tool for dealing with uncertain. In this paper, we extend it further by introducing the definition of a SB-bipolar interval neutrosophic set. Mathematical operations such as complement, union, and intersection are developed for these sets. Theorems related to SB-bipolar interval neutrosophic set are also proved.

Keywords: Bipolar neutrosophic sets, fuzzy set, interval neutrosophic sets, interval fuzzy set, neutrosophic sets.

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المستخلص:

دالة **النتروفزك Neutrosophic** من الدوال الرياضية التي تستخدم في دراسة البيانات المتعددة والغير دقيقة واستنادا علي هذه الدالة الرياضية تم تعريف دالة جديدة وهي اس بي النتروفزك ذات الفترة ثنائية القطب في هذه الورقة البحثية لتكون اوسع وأكثر شمولا في التعامل مع البيانات. كما تم دراسة الخصائص الرياضية لهذه الدالة واثبات العديد من النظريات المتعلقة بها.

الكلمات المفتاحية: الدالة نتروفزك ثنائية القطب، الدالة الضبابية، الدالة نترو فزك، الدالة الضبابية ذات الفترة، الدالة نتروفزك ذات الفترة.

1. Introduction:

In1965 Zadeh defined the novel concept of fuzzy set as a constructive tool which used widely in many applications involving uncertainty. (Atanassov 1986) extended the idea of fuzzy set theory to the intuitionistic fuzzy set, during which every element has both a membership

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degree and a non-membership degree. It's obvious that intuitionistic fuzzy sets are more useful than fuzzy set and complex bipolar fuzzy sets and multi q-fuzzy set (Fatma & Hassan 2016,2019, Fatma. & Basma 2023, Gulistan *et al.,* 2021) theory to deal the varied sorts of uncertainty model. (Smarandache 1999, Abdulkadhim *et al.,* 2022) defined the notion of neutrosophic set as novel mathematical framework which generalizes the concept of classical sets, fuzzy sets and interval valued fuzzy sets. Moreover, (Turksen 1986, Das *et. al,* 2020) extended the theory of neutrosophic fuzzy set and applied these theories in practical problems. Neutrosophic sets handle the indeterminate and inconsistent information that exists commonly in our daily life. In recent years, there have been an increasing number of studies on extended neutrosophic to algebraic theory. Based on these idea, (Wang *et al.,* 2010) studied single-valued neutrosophic sets in order to use them in scientific and engineering fields that give an additional possibility to represent uncertainty, incomplete, imprecise, and inconsistent data. On other hand, (Wang *et al.,* 2005), further the research on neutrosophic and define a new concept of interval neutrosophic to be used on interval neutrosophic theory. The hybrid of of interval fuzzy set and neutrosophic sets models was studied by (Lee 2000, Saad *et al.,* 2022). The aim of this paper is to develop a mathematical tool to solve uncertainty problem, namely SB-bipolar interval neutrosophic set which is a combination of interval fuzzy set, fuzzy set and bipolar interval neutrosophic set as a generalization of neutrosophic set. This set theory provides a bipolar interval-based membership structure to handle the SB-neutrosophic data.

2. Preliminaries:

In this section we recall the definition and basic operations on neutrosophic set and interval neutrosophic set.

1.2. Definition (Smarandache 1999). A neutrosophic set $Ƀ$ on U is $Ƀ = < T_{Ƀ}(x), I_{Ƀ}(x), F_{Ƀ}(x) >; x \in U$, where $T_{Ƀ}(x), I_{Ƀ}(x), F_{Ƀ}(x): Ƀ \to]-0, 1^{+}[$ and -0 < $T_{Ƀ}(x), I_{Ƀ}(x), F_{Ƀ}(x) < n3^{+}$.

2.2.Definition (Smarandache 1999). Let N_1 and N_2 be two neutrosophic sets on E. Then,

$$1 - N_1 \cap N_2 = < \min\left(T_{N_1}(x), T_{N_2}(x)\right), \max\left(I_{N_1}(x), I_{N_2}(x)\right), \max\left(F_{N_1}(x), F_{N_2}(x)\right) >.$$

$$2 - N_1 \cup N_2 = < \max\left(T_{N_1}(x), T_{N_2}(x)\right), \min\left(I_{N_1}(x), I_{N_2}(x)\right) \min\left(F_{N_1}(x), F_{N_2}(x)\right) >.$$

3.2.Definition (Delia *et al.,*2015)The bipolar neutrosophic set $Ƀ$ on U is

$Ƀ = < T_{Ƀ}^{+}(x), I_{Ƀ}^{+}(x), F_{Ƀ}^{+}(x),\ T_{Ƀ}^{-}(x), I_{Ƀ}^{-}(x),\ F_{Ƀ}^{-}(x) >; x \in U\}$ where $T_{Ƀ}^{+}(x), I_{Ƀ}^{+}(x), F_{Ƀ}^{+}(x): Ƀ \to [0, 1]$ and $T_{Ƀ}^{-}(x), I_{Ƀ}^{-}(x), F_{Ƀ}^{-}(x): Ƀ \to [-1,0]$.

The positive membership degree $T_{Ƀ}^{+}(x), I_{Ƀ}^{+}(x), F_{Ƀ}^{+}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in U$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T_{Ƀ}^{-}(x), I_{Ƀ}^{-}(x), F_{Ƀ}^{-}(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in U$ to some implicit counter-property corresponding to a bipolar neutrosophic set $Ƀ$.

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4.2.Definition (Wang *et al.,* 2005).An interval value neutrosophic set (IVN-sets) A in U is characterized by truth-membership function T_B, a indeterminacy-membership function I_B and a falsity-membership function F_B. For each point x ∈ U; $T_B(x), I_B(x), F_B(x) \subseteq [0,1]$.

5.2.Definition (Satya Narayana *et al.,* 2023). Let K be a non-empty set. An SB-neutrosophic set (SB-NSS) in K is a structure of the form

$N =< \zeta, \tilde{\alpha}_t(\zeta), \alpha_i(\zeta), \alpha_f(\zeta): \zeta \in K >$, where α_i and α_f are fuzzy set in K; which are called a degree of indeterminacy and degree of non-membership, respectively. $\tilde{\alpha}_t$ is an interval value fuzzy set in K, which is called an interval valued degree of membership.

3. SB-Bipolar Interval Neutrosophic Set:

With some examples and results, we give the concept of SB-bipolar interval neutrosophic set.

1.3. Definition Let Ų be a non-empty set. An SB-bipolar interval neutrosophic set (SB-BINSS) in Ų is a structure of the form

$$Ƀ_{BI} =< \zeta, [[\tilde{\alpha}_t(\zeta)^l, \tilde{\alpha}_t(\zeta)^u], [\tilde{\alpha}_t(\zeta)^l, \tilde{\alpha}_t(\zeta)^u]]^+, [\alpha_i(\zeta)^l, \alpha_i(\zeta)^u]^+, [\alpha_f(\zeta)^l, \alpha_f(\zeta)^u]^+,$$

$$[[\tilde{\alpha}_t(\zeta)^l, \tilde{\alpha}_t(\zeta)^u], [\tilde{\alpha}_t(\zeta)^l, \tilde{\alpha}_t(\zeta)^u]]^-, [\alpha_i(\zeta)^l, \alpha_i(\zeta)^u]^-, [\alpha_f(\zeta)^l, \alpha_f(\zeta)^u]^-: \zeta \in Ų >,$$

where $\alpha_i^+, \alpha_i^-, \alpha_f^+, \alpha_f^-: Ų \rightarrow [0,1]$are fuzzy set in U; which are called a degree of indeterminacy and degree of non-membership, respectively.

$\tilde{\alpha}_t^+, \tilde{\alpha}_t^-: U \rightarrow [-0,-1]$is an interval value fuzzy set in U, which is called an interval valued degree of membership.

Example Let $Ų = \{u_1, u_2\}$ be the universe of discourse where u_1 is characterises the shape, u_2 indicates the weight of the objects. It may be further assumed that the values of u_1 and u_2 are subset of [0, 1] and they are obtained from an expert person. The expert construct an SB-bipolar interval neutrosophic set the characteristics of the objects according to by membership function $\alpha_i^+, \alpha_i^-, \alpha_f^+, \alpha_f^-$and non-membership function $\tilde{\alpha}_t^+, \tilde{\alpha}_t^-$, as follows;

$$N_{BI} = \{u_1, [[0.1,0.3], [0.5, 0.7]]^+, [0.3,0.5]^+, [0,0.4]^+, [[-0.3,-0.1],$$

$$[-0.7,-0.5]]^-, [-0.5,-0.2]^-, [-0.2,-0.1]^-, u_2, [[0.1,0.2], [0.4, 0.7]]^+,$$

$$[0.3,0.4]^+, [0,0.3]^+, [[-0.1,-0.1],[-0.7,-0.5]]^-, [-0.5,-0.2]^-, [-0.2,-0.1]^-\}.$$

2.3.Definition Let Ƀ be an SB-bipolar interval neutrosophic set. Then for all $u \in$ Ų,

1. **Ƀ is empty, denoted by:**

$$\emptyset N_{BI} = \{u, [[0,0], [0, 0]]^+, [1,1]^+, [1,1]^+, [[-0,-0], [-0,-0]]^-, [-1,-1]^-, [-1,-1]^-: u \in Ų\}.$$

2. **Ƀ is universal denoted by:**

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$\breve{\mathrm{N}}_{BI} = \{u, [[1,1],[1,1]]^{+}, [0,0]^{+}, [0,0]^{+}, [[-1,-1],[-1,-1]]^{-}, [-0,-0]^{-}, [-0,-0]^{-} : u \in \text{Ų}\}$.

3. **The complement of Ƀ is denoted by $\overline{\text{Ƀ}}$ and is defined by:**

$\overline{\text{Ƀ}} = \{[[1-\tilde{\alpha}_t(u)^l, 1-\tilde{\alpha}_t(u)^u], [1-\tilde{\alpha}_t(u)^l, 1-\tilde{\alpha}_t(u)^u]^{+}], [1-\alpha_i(u)^l, 1-\alpha_i(u)^u]^{+}, [1-\alpha_f(u)^l, 1-\alpha_f(u)^u]^{+}, [[1-\tilde{\alpha}_t(u)^l, 1-\tilde{\alpha}_t(u)^u], [1-\tilde{\alpha}_t(u)^l, 1-\tilde{\alpha}_t(u)^u]]^{-}, [1-\alpha_i(u)^l, 1-\alpha_i(u)^u]^{-}, [1-\alpha_f(u)^l, 1-\alpha_f(u)^u]^{-} : u \in \text{Ų}\}$.

3.3. Definition An SB-bipolar interval neutrosophic set Ƀ_{a} is contained in the other SB-bipolar interval neutrosophic set Ƀ_{b} , such that $\text{Ƀ}_{a} \tilde{\subseteq} \text{Ƀ}_{b}$ if and only if:

$\text{Ƀ}_{a_{\tilde{\alpha}_t}}{}^{+} \le \text{Ƀ}_{b_{\tilde{\alpha}_t}}{}^{+}$, $\text{Ƀ}_{a_{\alpha_i}}{}^{+} \le \text{Ƀ}_{2_{\alpha_i}}{}^{+}$, $\text{Ƀ}_{a_{\alpha_f}}{}^{+} \ge \text{Ƀ}_{2_{\alpha_f}}{}^{+}$,

$\text{Ƀ}_{a_{\tilde{\alpha}_t}}{}^{-} \ge \text{Ƀ}_{2_{\tilde{\alpha}_t}}{}^{-}$, $\text{Ƀ}_{a_{\alpha_i}}{}^{-} \ge \text{Ƀ}_{a_{\alpha_i}}{}^{-}$ and $\text{Ƀ}_{a_{\alpha_f}}{}^{-} \le \text{Ƀ}_{2_{\alpha_f}}{}^{-}$.

Example Let $U = \{u_1, u_2\}$.Then Ƀ_{a}

$$= \begin{Bmatrix} u_1, [[0,0.1],[0.3,0.3]], [0,0.4], [0.8,0.9], [[-0.2,0],[-0.3,-0.2]], [-0.3,-0.1], [-0.4,0] \\ u_2, [[0,0.2],[0.3,0.4]], [0.1,0.3], [0,0.5], [[-0.2,0],[-0.5,-0.2]], [-0.2,0], [-0.4,-0.3] \end{Bmatrix}$$

Ƀ_{b}

$$= \begin{Bmatrix} u_1, [[0,0.3],[0.3,0.4]], [0,0.5], [0.5,0.7], [[-0.3,0],[-0.4,-0.2]], [-0.4,-0.2], [-0.4,0] \\ u_2, [[0,0.3],[0.4,0.4]], [0.2,0.5], [0,0.5], [[-0.2,0],[-0.6,-0.3]], [-0.3,0], [-0.2,-0.1] \end{Bmatrix}$$

are SB-bipolar interval neutrosophic set in U and $\text{Ƀ}_{a} \tilde{\subseteq} \text{Ƀ}_{b}$.

Thoreme Let Ƀ_{a} and Ƀ_{b} are SB-bipolar interval neutrosophic sets. If $\text{Ƀ}_{a} \tilde{\subseteq} \text{Ƀ}_{b}$ then $\overline{\text{Ƀ}_{b}} \tilde{\subseteq} \overline{\text{Ƀ}_{a}}$.

Proof. From Definitions 3.2 and 3.3 we have, $\text{Ƀ}_{a_{\tilde{\alpha}_t}}{}^{+} \le \text{Ƀ}_{b_{\tilde{\alpha}_t}}{}^{+}$, take the complement of each element

$[1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^u], [1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^u]^{+}$,

$[1-A_{2_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{b_{\tilde{\alpha}_t}}(u)^u], [1-\text{Ƀ}_{b_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{b_{\tilde{\alpha}_t}}(u)^u]^{+}$, , this implies

$[1-\text{Ƀ}_{b_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{b_{\tilde{\alpha}_t}}(u)^u]^{+} \le [1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^l, 1-\text{Ƀ}_{a_{\tilde{\alpha}_t}}(u)^u]^{+}$, then

$\overline{\text{Ƀ}_{b_{\tilde{\alpha}_t}}}{}^{+} \le \overline{\text{Ƀ}}_{a_{\tilde{\alpha}_t}}{}^{+}$

$[1-\text{Ƀ}_{a_{\alpha_i}}(u)^l, 1-\text{Ƀ}_{a_{\alpha_i}}(u)^u], [1-\text{Ƀ}_{b_{\alpha_i}}(u)^l, 1-\text{Ƀ}_{b_{\alpha_i}}(u)^u]^{+}$, then

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$[1 - Ƀ_{ƀ_{\alpha_i}}(u)^l, 1 - Ƀ_{ƀ_{\alpha_i}}(u)^u]^+ \leq [1 - Ƀ_{a_{\alpha_i}}(u)^l, 1 - Ƀ_{a_{\alpha_i}}(u)^u]$,

$\overline{Ƀ}_{ƀ_{\alpha_i}}{}^+ \leq \overline{Ƀ}_{a_{\alpha_i}}{}^+$,

$[1 - Ƀ_{a_{\alpha_f}}(u)^l, 1 - Ƀ_{a_{\alpha_f}}(u)^u], [1 - Ƀ_{ƀ_{\alpha_f}}(u)^l, 1 - Ƀ_{ƀ_{\alpha_f}}(u)^u]^+$, then

$[1 - Ƀ_{ƀ_{\alpha_f}}(u)^l, 1 - Ƀ_{ƀ_{\alpha_f}}(u)^u]^+ \leq \left[1 - Ƀ_{a_{\alpha_f}}, 1 - Ƀ_{a_{\alpha_f}}(u)^u\right]$, then $\overline{Ƀ}_{ƀ_{\alpha_f}} \leq \overline{Ƀ}_{a_{\alpha_f}}$.

In similar way we can get

$Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^- \geq Ƀ_{a_{\tilde{\alpha}_t}}{}^-$, $Ƀ_{ƀ_{\alpha_i}}{}^- \geq Ƀ_{a_{\alpha_i}}{}^-$and $Ƀ_{ƀ_{\alpha_f}}{}^- \leq Ƀ_{a_{\alpha_f}}{}^-$. Since

$\overline{Ƀ}_{ƀ_{\tilde{\alpha}_t}}{}^+ \leq \overline{Ƀ}_{a_{\tilde{\alpha}_t}}{}^+$, $\overline{Ƀ_{ƀ_{\alpha_i}}}{}^+ \leq \overline{Ƀ}_{a_{\alpha_i}}{}^+$, $\overline{Ƀ}_{ƀ_{\alpha_f}}{}^+ \geq \overline{Ƀ}_{a_{\alpha_f}}{}^+$,

$\overline{Ƀ}_{ƀ_{\tilde{\alpha}_t}}{}^- \geq \overline{Ƀ}_{a_{\tilde{\alpha}_t}}{}^-$, $\overline{Ƀ}_{ƀ_{\alpha_i}}{}^- \geq \overline{Ƀ}_{a_{\alpha_i}}{}^-$and $\overline{Ƀ}_{ƀ_{\alpha_f}}{}^- \leq \overline{Ƀ}_{a_{\alpha_f}}{}^-$, therefor $\overline{Ƀ}_{ƀ} \subseteq \overline{Ƀ}_{a}$.

Theorem 3.2. SB-bipolar interval neutrosophic set is the generalization of a bipolar cubic fuzzy set.

Proof. Suppose that *A* is a SB-bipolar interval neutrosophic set. Then by setting the positive components $\alpha_i{}^+$, $\alpha_f{}^+$ equals to zero as well as the negative components $\alpha_i{}^-$, $\alpha_f{}^-$ equals to zero reduces the SB-bipolar interval neutrosophic set is the generalization of a bipolar cubic fuzzy set.

Definition 3.4 Let $Ƀ_a$ and $Ƀ_ƀ$ be two SB-bipolar interval neutrosophic set on U. Then,

$1 - Ƀ_a \cap Ƀ_ƀ =< (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+L}\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+U})]$,

$[\min Ƀ_{a_{\alpha_i}}{}^{+L}, Ƀ_{ƀ_{\alpha_i}}{}^{+L}), \min(Ƀ_{a_{\alpha_i}}{}^{+U} + Ƀ_{ƀ_{\alpha_i}}{}^{+U})], [\max\left(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+L}\right)$,

$\max\left(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}\right), (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-L}\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-U})]$,

$\left[\max(Ƀ_{a_{\alpha_i}}{}^{-L}, Ƀ_{ƀ_{\alpha_i}}{}^{-L}\right), \max(Ƀ_{a_{\alpha_i}}{}^{-U}, Ƀ_{ƀ_{\alpha_i}}{}^{-U})], [\min\left(Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{ƀ_{\alpha_f}}{}^{-L}\right), \min\left(Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{ƀ_{\alpha_f}}{}^{-U}\right)]$ >.

$2 - Ƀ_a \cup Ƀ_ƀ =< (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+L}\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+U})]$,

$\left[\max Ƀ_{a_{\alpha_i}}{}^{+L}, Ƀ_{ƀ_{\alpha_i}}{}^{+L}\right), \max(Ƀ_{a_{\alpha_i}}{}^{+U}, Ƀ_{ƀ_{\alpha_i}}{}^{+U})], [\min\left(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+L}\right)$,

$\min\left(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}\right), (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-L}\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-U})]$,

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$\left[\min\left(Ƀ_{a_{\alpha_i}}{}^{-L}, Ƀ_{b_{\alpha_i}}{}^{-L}\right), \min\left(Ƀ_{a_{\alpha_i}}{}^{-U}, Ƀ_{b_{\alpha_i}}{}^{-U}\right)\right], \left[\max\left(Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{b_{\alpha_f}}{}^{-L}\right), \max\left(Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{b_{\alpha_f}}{}^{-U}\right)\right]$ >.

Example 3.3. From **Example 3.2** we have:

$1 -\ Ƀ_a \cap Ƀ_b =< u_1$, ([0,0.1] , [0.3,0.3]), [0 ,0.4], [0.8,0.9], ([-0.2,0], [-0.3, -0.2]), [-0.3,-0.1] $[-0.4,0]), u_2$([0,0.2],[0.3,0.4]),[0.15,0.4],[0.1,0.3],([- 0.2,-0] [-0.5, -0.2]), [-0.2, -0], [-0.4, - 0. 3] >.

$2 -\ Ƀ_a \cup Ƀ_b =< u_1([0,0.3],[0.3,0.4])[0,0.5], [0.0.5], ([-0.3,0],[-0.4,-0.2])$ $[-0.4,-0],[-0.4,0], u_2$, ([0,0.3], [0.4,0.4]) , [0.2,0.5], [0,0.5], ([-0.3, -0], [-0.6, -0.3]),[-0.35,-0],[-0.2,-0.1] >.

Theorem 3.3 Let $Ƀ_a$ and $Ƀ_b$ be two SB-bipolar interval neutrosophic sets on U. Then $Ƀ_a \cup Ƀ_b$ is the largest SB-bipolar interval neutrosophic set containing both $Ƀ_a$ and $Ƀ_b$.

Proof. From Definition 3.4 we have,

$Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}$ and $Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L} \le (Ƀ_a \cup Ƀ_b)_{\tilde{\alpha}_t{}^{+L}} = \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L})$,

$Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}$ and $Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U} \le Ƀ_a \cup Ƀ_b)_{\tilde{\alpha}_t{}^{+U}} = \max(Ƀ_{a_{\tilde{\alpha}_t\tilde{\alpha}_t}}{}^{+U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U})$

$Ƀ_{a_{\tilde{\alpha}_t\alpha_i}}{}^{+L}$ and $Ƀ_{b_{\alpha_i}}{}^{+L} \le (Ƀ_{a_{\tilde{\alpha}_t}} \cup Ƀ_b)_{\alpha_i{}^{+L}} = \max\left(A_{1_{\alpha_i}}{}^{+L}, Ƀ_{b_{\alpha_i}}{}^{+L}\right)$,

$Ƀ_{a_{\tilde{\alpha}_t\alpha_i}}{}^{+U}$ and $Ƀ_{b_{\alpha_i}}{}^{+U} \le (Ƀ_{a_{\tilde{\alpha}_t}} \cup Ƀ_b)_{\alpha_i{}^{+U}} = \max\left(A_{1_{\alpha_i}}{}^{+U}, Ƀ_{b_{\alpha_i}}{}^{+U}\right)$,

$Ƀ_{a_{\tilde{\alpha}_t\alpha_f}}{}^{+L}$ and $Ƀ_{b_{\alpha_f}}{}^{+L} \ge \min\left(Ƀ_{a_{\tilde{\alpha}_t\alpha_f}}{}^{+L}, Ƀ_{b_{\alpha_f}}{}^{+L}\right)$,

$Ƀ_{a_{\tilde{\alpha}_t\alpha_f}}{}^{+U}$ and $Ƀ_{b_{\alpha_f}}{}^{+U} \ge \min\left(Ƀ_{a_{\tilde{\alpha}_t\alpha_f}}{}^{+U}, Ƀ_{b_{\alpha_f}}{}^{+U}\right)$,

$Ƀ_{a_{\tilde{\alpha}_t\tilde{\alpha}_t}}{}^{-L}$ and $Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L} \ge Ƀ_{a_{\tilde{\alpha}_t\tilde{\alpha}_t}}{}^{-L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L})$,

$Ƀ_{a_{\tilde{\alpha}_t\tilde{\alpha}_t}}{}^{-U}$ and $Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U} \ge \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U})]$,

$Ƀ_{a_{\tilde{\alpha}_t\alpha_i}}{}^{-L}$ and $Ƀ_{b_{\alpha_i}}{}^{-L} \ge \min\left(Ƀ_{a_{\alpha_i}}{}^{-L}, Ƀ_{b_{\alpha_i}}{}^{-L}\right)$,

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$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}_{\alpha_i}{}^{-U}$ and $Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U} \geq \min\left(Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U}\right)$,

$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}_{\alpha_f}{}^{-L}$ and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L} \leq \max\left(Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{-L}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{-U}$and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U} \leq \max\left(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}_{\alpha_f}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U}\right)$.

Therefore $(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}} \cup Ƀ_{\mathfrak{b}})$is the largest SB-bipolar interval neutrosophic set containing both $Ƀ_{\mathfrak{a}}$ and $Ƀ_{\mathfrak{b}}$.

Theorem 3.4 Let $Ƀ_{\mathfrak{a}}$and $Ƀ_{\mathfrak{b}}$ be two SB-bipolar interval neutrosophic sets on U. Then $Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}}$is the smallest SB-bipolar interval neutrosophic set containing both $Ƀ_{\mathfrak{a}}$ and $Ƀ_{\mathfrak{b}}$.

Proof. From **Definition 3.4** we have,

$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{+L}$and $Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+L} \geq (Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}})_{\tilde{\alpha}_t{}^{+L}} = \min(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+L})$,

$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{+U}$ and $Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+U} \geq (Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}})_{\tilde{\alpha}_t{}^{+U}} = \min(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+U})$,

$Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{+L}$ and $Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+L} \geq (Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}})_{\alpha_i{}^{+L}} = \min\left(Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{+L}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+L}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{+U}$ and $Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+U} \geq (Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}})_{\alpha_i{}^{+U}} = \min\left(Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{+U}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+U}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{+L}$ and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+L} \leq \max\left(Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{+L}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+L}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{+U}$ and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+U} \leq \max\left(Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{+U}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+U}\right)$,

$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{-L}$and $Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L} \leq \max(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L})$,

$Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{-U}$ *and* $Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U} \leq \max(Ƀ_{\mathfrak{a}_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U})]$,

$Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{-L}$ and $Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-L} \leq \max\left(Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{-L}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-L}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{-U}$ and $Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U} \leq \max\left(Ƀ_{\mathfrak{a}_{\alpha_i}}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U}\right)$,

$Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{-L}$ and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L} \geq \min\left(Ƀ_{\mathfrak{a}_{\alpha_f}}{}^{-L}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L}\right)$,

$A_{1_{\alpha_f}}{}^{-U}$and $Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U} \geq \min\left(A_{1_{\alpha_f}}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U}\right)$. Therefore $(Ƀ_{\mathfrak{a}} \cap Ƀ_{\mathfrak{b}})$is the largest SB-bipolar interval neutrosophic set containing both $Ƀ_{\mathfrak{a}}$ and $Ƀ_{\mathfrak{b}}$.

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Definition 3.5. Addition of $Ƀ_{a}$ and $Ƀ_{b}$, denoted by $A_1 + A_2$

$=< \left[\min\left(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L}, 1\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U}, 1)\right], [\min\left(Ƀ_{a_{\alpha_i}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L}, 1\right), \min Ƀ_{a_{\alpha_i}}{}^{+U} + Ƀ_{b_{\alpha_i}}{}^{+U}, 1)], [\min\left(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{b_{\alpha_f}}{}^{+L}, 1\right)$

$\min(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{b_{\alpha_f}}{}^{+U})], \left[\max\left(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L}, -1\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U}, -1)\right], \max\left(Ƀ_{a_{\alpha_i}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L}, -1\right), [\max\left(Ƀ_{a_{\alpha_i}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L}, -1\right),$

$\max(Ƀ_{a_{\alpha_i}}{}^{-U} + Ƀ_{b_{\alpha_i}}{}^{-U}, 1)], [\max\left(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{b_{\alpha_f}}{}^{-L}, 1\right), \max(Ƀ_{a_{\alpha_f}}{}^{-U}$

$+ Ƀ_{b_{\alpha_f}}{}^{-U}, 1)] >.$

Definition 3.6. Difference of $Ƀ_{a}$ and $Ƀ_{b}$, denoted by $\frac{Ƀ_{a}}{Ƀ_{b}}$, is defined by

$=< (\left[\min(Ƀ_{a\ \tilde{\alpha}_t}{}^{+L}, Ƀ_{b_{\alpha_f}}{}^{+L}\right), \min(Ƀ_{a\ \tilde{\alpha}_t}{}^{+U}, Ƀ_{b_{\alpha_f}}{}^{+U})], [\max\left(Ƀ_{a\ \alpha_i}{}^{+L}, 1 - Ƀ_{b_{\alpha_i}}{}^{+U}\right),$
$\max(Ƀ_{a\ \alpha_i}{}^{+U}, 1 - Ƀ_{b_{\alpha_i}}{}^{+L})], [\max\left(Ƀ_{a\ \alpha_f}{}^{+L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L}\right),$

$\max(Ƀ_{a\ \alpha_f}{}^{+U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U})], (\left[\max(Ƀ_{a\ \tilde{\alpha}_t}{}^{-L}, Ƀ_{b_{\alpha_f}}{}^{-L}\right), \max(Ƀ_{a\ \tilde{\alpha}_t}{}^{-U}, Ƀ_{b_{\alpha_f}}{}^{-U})],$

$[\min\left(Ƀ_{a\ \alpha_i}{}^{-L}, 1 - Ƀ_{b_{\alpha_i}}{}^{-U}\right), \min(Ƀ_{a\ \alpha_i}{}^{-U}, 1 - Ƀ_{b_{\alpha_i}}{}^{-L})],$

$[\min\left(Ƀ_{a\ \alpha_f}{}^{-L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L}\right), \min(Ƀ_{a\ \alpha_f}{}^{-U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U})] >.$

Example 3.4. From **Example 3.2** $\frac{Ƀ_{a}}{Ƀ_{b}}$ is

$=< u_1, [[0,0.1], [0.3,0.3]], [0,0.5], [0.5,0.7], [[-0.4,0], [-0.4,-0.2]]$

$[-0.6,-0.8], [-0.4,-0.2]\ u_2, [[0,0.2], [0,0.4]], [0.8,0.5], [0.3,0.5],$

$[[-0.2,-0.1], [-0.5,-0.2]], [-0.7,0], [-0.5,-0.2] >.$

Definition 3.7. Scalar multiplication of $Ƀ_{a}$ denoted by $Ƀ_{a*a}$ is defined by

$=< (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} * a, [1,1]\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} * a, [1,1])], [\min\left(Ƀ_{a_{\alpha_i}}{}^{+L} * a, 1\right),$

$\min Ƀ_{a_{\alpha_i}}{}^{+U} * a, 1)], [\min\left(Ƀ_{a_{\alpha_f}}{}^{+L} * a, 1\right), \min(Ƀ_{a_{\alpha_f}}{}^{+U} * a, 1)],$

$\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} * a, [0,0]\right) \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} * a, [0,0])], [\max\left(Ƀ_{a_{\alpha_i}}{}^{-L} * a, 0\right),$

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$\max(Ƀ_{a_{\alpha_i}}{}^{-U} * a, 0)], [\max\left(Ƀ_{a_{\alpha_f}}{}^{-L} * a, 0\right), \max(Ƀ_{a_{\alpha_f}}{}^{-U} * a, 0)] >.$

Example 3.5. Let $Ų = \{u_1, u_2\}$ and $Ƀ_{a}$

$= u_1, [[0,0.3], [0.1,0.3]], [0,0.4], [0.6,0.9], [[-0.2,0], [-0.4,-0.2]],$

$[-0.5,-0.1], [-0.4,0]\ u_2, [[0,0.1], [0.3,0.6]], [0.1,0.3], [0,0.6],$

$[-0.4,0], [-0.4,-0.3].$

Then $Ƀ_{a*2} =$

$$= \begin{Bmatrix} u_1, [[0,0.6], [0.2,0.6]], [0,0.8], [1,1], [[0,0], [0,0]], [0,0], [0,0] \\ u_2, [[0,0.1], [0.6,1]], [0.2,0.6], [0.2,0.6], [[0,0], [0,0]], [0,0], [0,0] \end{Bmatrix}.$$

Definition 3.8 Truth-Favorite of $Ƀ_{a}$, denoted by $\Delta Ƀ_{a}$, is defined by

$=< (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{a_{\alpha_i}}{}^{+U}, 1)],$

$[0,0], [Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{a_{\alpha_f}}{}^{+U}], (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, -1\right),$

$\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, -1)], [-0,-0], [Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{a_{\alpha_f}}{}^{-U}]) >.$

Definition 3.9 False -Favorite of $Ƀ_{a}$, denoted by $\nabla Ƀ_{a}$, is defined by

$=< \left[Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}\right], [0,0], \min(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1) \min(Ƀ_{a_{\alpha_f}}{}^{+U}$

$+ Ƀ_{a_{\alpha_i}}{}^{+U}, 1)], \left[Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}\right], [-0,-0], \max(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, -1)$

$\max(Ƀ_{a_{\alpha_f}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, -1)] >.$

Example 3.6. Let $U = \{u_1, u_2\}$.Then $Ƀ_{a}$

$= \{u_1, ([[0,0.2], [0.2,0.3]], [0,0.4], [0.7,0.9], [[-0.2,-0], [-0.3,-0.2]],$

$[-0.3,-0.1], [-0.4,0]), u_2, [[0,0.2], [0.3,0.4]], [0.1,0.3], [0,0.5], [[-0.2,-0],$

$[-0.5,-0.2]], [-0.2,-0], [-0.4,-0.3])\}.$

Then

$\Delta Ƀ_{a} = \{u_1, [[\min(0+0,1), \min(0.2+0.4,1)], [\min(0.2+0,1), \min(0.3$

$+0.4), 1)]]\ [0,0], [0.7,0.9], [\max(-0.2-0.3,-1), \max(0-0.1,-1)],$

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$[\max(-0.3-0.1,-1), \max(-0.2-0.1,-1)], [-0,-0], [-0.4,0]$,

$u_2, ([\min(0+0,1), \min(0.2+0.3,1)], [\min(0.3+0.1,1), \min(0.4$

$+0.3),1)], [0,0] \max(-0.5-0.2,-1), \max(-0.2-0,-1)], [-0,-0]$,

$[-0.4,-0.3]\}$,

$= \{u_1, ([0, 0.6], [0.2,0.7]), [0,0], [0.7,0.9], ([-0.5, -0], [-0.4, -0.3]), [-0, -0]$,

$[-0.4,0]), u_2, [0,0.5], [0.4,0.7], [0,0], [0,0.5], ([-0.4,-0.7],[-0.7,-0.2]$

And $\nabla Ƀ_{a}$

$=< u_1, \big[[0,0.2], [0.2,0.3]\big], [0,0], [\min(0.7+0,1,1), \min(0.9+0.4,1)]$,

$\big[[-0.2,0], [-0.3,-0.2]\big], [-0,-0], [\max(-0.4-0.3), -1) \max(0-0.1), -1)])$,

$\big[[0,0.2], [0.3,0.4]\big], [0,0], [\min(0+0,1,1), \min(0.5+0.3,1)]$,

$\big[[-0.2,0], [-0.5,-0.2]\big], [-0,-0], [\max(-0.4-0.2), -1) \max(0.3-0), -1)] >$,

$= \big[[0,0.2], [0.2,0.3]\big], [0,0], [0.8,1], \big[[-0.2,0], [-0.3,-0.2]\big], [-0,-0], [-0.7,-0.1])$,

$\big[[-0.2,0], [-0.3,-0.2]\big], [-0,-0], [-0.7,-1), -0.1] >$.

Theorem3.5. Let $\Delta Ƀ_{a}$ and $\Delta Ƀ_{ƀ}$ are SB-bipolar interval neutrosophic set. Then

1) $\Delta \Delta Ƀ_{a} = \Delta Ƀ_{a}$.

2) $\Delta (Ƀ_{a} \cap Ƀ_{ƀ}) \subseteq \Delta Ƀ_{a} \cap \Delta Ƀ_{ƀ}$.

3) $\Delta (Ƀ_{a} \cup Ƀ_{ƀ}) \subseteq \Delta Ƀ_{a} \cup \Delta Ƀ_{ƀ}$.

Proof. 1) The proofs can be easily obtained from Definition 3.6.

2) Using Definition 3.4 and 3.6 then,

$\Delta (Ƀ_{a} \cap Ƀ_{ƀ}) = < (\Big[\min(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+L}\Big) + \min(Ƀ_{a_{\alpha_i}}{}^{+L}, Ƀ_{ƀ_{\alpha_i}}{}^{+L})$,

$1)), \mathrm{mim}(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+U}) + \min(Ƀ_{a_{\alpha_i}}{}^{+U}, Ƀ_{ƀ_{\alpha_i}}{}^{+U}),1], [0,0]$,

$[\max\Big(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+L}\Big), \max\Big(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}\Big)], [\max(\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}$,

$Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}) + \max(Ƀ_{a_{\alpha_i}}{}^{-L}, Ƀ_{ƀ_{\alpha_i}}{}^{-L}), -1), \max(\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-U})$

$+\max(Ƀ_{a_{\alpha_i}}{}^{-U}, Ƀ_{ƀ_{\alpha_i}}{}^{-U}), -1)], [0,-0], [\max\Big(Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{ƀ_{\alpha_f}}{}^{-L}\Big)$,

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$\max\left((Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{b_{\alpha_f}}{}^{-U}\right)] >.$

Implies that

$\subseteq\left[\min(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1)\min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L}, 1\right)), \min \quad (\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{a_{\alpha_i}}{}^{+U}, 1), \min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{b_{\alpha_i}}{}^{+U}, 1))], [0,0]$

$\left[\max(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{a_{\alpha_f}}{}^{+L}\right), \max(Ƀ_{b_{\alpha_f}}{}^{+L}, Ƀ_{b_{\alpha_f}}{}^{+L})],$

$(\left[\max(\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, 1), (\max(Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L}, 1\right)),$

$\max(\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{a_{\alpha_i}}{}^{-U} - 1)], \max(A_{2_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{b_{\alpha_i}}{}^{-U}, -1))], [0,0], \left[\max Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{b_{\alpha_f}}{}^{-L}\right),$

$\max\ (Ƀ_{b_{\alpha_f}}{}^{-U}, Ƀ_{b_{\alpha_f}}{}^{-U})] >,$

$\subseteq< (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}+, Ƀ_{a_{\alpha_i}}{}^{+U}, 1)], [0,0], [Ƀ_{a_{\alpha_f}}{}^{+L},$

$Ƀ_{a_{\alpha_f}}{}^{+U}], (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, -1\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, -1)], [-0, -0],$

$[Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{a_{\alpha_f}}{}^{-U}]) >, \cap< (\left[\min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L}, 1\right),)], [0,0],$

$[Ƀ_{b_{\alpha_f}}{}^{+L}, Ƀ_{b_{\alpha_f}}{}^{+U}], \left[\min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L}, -1\right), \min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U}+, Ƀ_{b_{\alpha_i}}{}^{-U}, -1)],$

$[-0, -0],\ [Ƀ_{b_{\alpha_f}}{}^{-L}, Ƀ_{b_{\alpha_f}}{}^{-U}]) >.$ Therefore $\subseteq \Delta Ƀ_a \cap \Delta Ƀ_b$.

3) We can prove it in a similar way in 2 using Definition 3.4 and 3.6 such that,

$\Delta (Ƀ_a \cup Ƀ_b) = < (\left[\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L}\right) + \min(Ƀ_{a_{\alpha_i}}{}^{+L}, Ƀ_{b_{\alpha_i}}{}^{+L}), 1),$

$\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U}) + \min(Ƀ_{a_{\alpha_i}}{}^{+U}, Ƀ_{b_{\alpha_i}}{}^{+U}), 1], [0,0], [\min$

$\left(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{b_{\alpha_f}}{}^{+L}\right), \min\left(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{b_{\alpha_f}}{}^{+U}\right)], [\min(\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}) +$

$\max(Ƀ_{a_{\alpha_i}}{}^{-L}, Ƀ_{b_{\alpha_i}}{}^{-L}), -1), \min(\max\left(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U}\right) \quad + \max\left(Ƀ_{a_{\alpha_i}}{}^{-U} + Ƀ_{b_{\alpha_i}}{}^{-U}\right),$
$-1)], [-0, -0], [\max\left(Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{b_{\alpha_f}}{}^{-L}\right), \max\left(Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{b_{\alpha_f}}{}^{-U}\right)] >$, then

$\subseteq\left[\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1), \min(Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L}, 1\right)),$

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$\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{a_{\alpha_i}}{}^{+U} 1), \min(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{ƀ_{\alpha_i}}{}^{+U}, 1))], [0,0],,$

$\left[\left(\min\left((Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}\right)\right], \min(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+L}\right),$

$\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + A_{1_{\alpha_i}}{}^{-L}, 1), \max(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-L} Ƀ_{ƀ_{\alpha_i}}{}^{-L}, 1)),$

$\max(\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, -1], (\min(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{ƀ_{\alpha_i}}{}^{-U}, -1))],$

$[0,0], \left[\min\ (Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{ƀ_{\alpha_f}}{}^{-L}\right), \min\ (Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{ƀ_{\alpha_f}}{}^{-U})] >,$

$\subseteq < (\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{a_{\alpha_i}}{}^{+U}, 1)], [0,0],$

$[Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{a_{\alpha_f}}{}^{+U}], (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, -1\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} +, Ƀ_{a_{\alpha_i}}{}^{-U}, -1)],$

$[-1, -1], (Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{a_{\alpha_f}}{}^{-U}]) >,$

$\cup < (\left[\min(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{ƀ_{\alpha_i}}{}^{+L}, 1\right),)], [0,0], [Ƀ_{ƀ_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}], ([(\max(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-L}$

$+ Ƀ_{ƀ_{\alpha_i}}{}^{-L}, -1), \max(Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{ƀ_{\alpha_i}}{}^{-U}, -1)], [-0, -0], Ƀ_{ƀ_{\alpha_f}}{}^{-L}, Ƀ_{ƀ_{\alpha_f}}{}^{-U}]) >,$

thus $\subseteq \Delta Ƀ_a \cup \Delta Ƀ_ƀ$.

Theorem3.6. Let $\nabla Ƀ_a$and $\nabla Ƀ_ƀ$ are SB-bipolar interval neutrosophic set. Then

1) $\nabla \nabla Ƀ_a = \nabla Ƀ_a$.

2) $\nabla (Ƀ_a \cap Ƀ_ƀ) \subseteq \nabla Ƀ_a \cap \nabla Ƀ_ƀ$.

3) $\nabla (Ƀ_a \cup Ƀ_ƀ) \subseteq \nabla Ƀ_a \cup \nabla Ƀ_ƀ$.

4) $\nabla(Ƀ_a + Ƀ_ƀ) = \nabla Ƀ_a + \nabla Ƀ_ƀ$.

Proof.1) The proofs can be obtained from Definition3.6.

2) Using Definition3.4 and 3.6 we have

$\nabla (Ƀ_a \cap Ƀ_ƀ) =$

$< (\left[\mathit{min}(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+L}\right), \mathit{min}(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{ƀ_{\tilde{\alpha}_t}}{}^{+U})], [0,0], [\max(\min\left(Ƀ_{a_{\alpha_f}}{}^{+L}, Ƀ_{ƀ_{\alpha_f}}{}^{+L}\right)$

$+\mathit{min}(Ƀ_{a_{\alpha_i}}{}^{+L}, Ƀ_{ƀ_{\alpha_i}}{}^{+L}), 1), \max\left(\min\left(Ƀ_{a_{\alpha_f}}{}^{+U}, Ƀ_{ƀ_{\alpha_f}}{}^{+U}\right) + \min(Ƀ_{a_{\alpha_i}}{}^{+U}, Ƀ_{ƀ_{\alpha_i}}{}^{+U}\right),$

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$1),] \left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L}\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U})], [-0,-0], [\min(\max\left((Ƀ_{a_{\alpha_f}}{}^{-L}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L}\right) + \max((Ƀ_{a_{\alpha_i}}{}^{-L}, + (Ƀ_{a_{\alpha_i}}{}^{-L}, -1) \min(\max\left((Ƀ_{a_{\alpha_f}}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U}\right) \max\left((Ƀ_{a_{\alpha_i}}{}^{-U}, Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U}\right), -1)] >$. Implies that

$\subseteq (\left[min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+L}\right), min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+U})], [0,0], [\max(min(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1), \min(A_{2_{\alpha_f}}{}^{+L} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+L}, 1), \max(\min(Ƀ_{a_{\alpha_f}}{}^{+U} + (Ƀ_{a_{\alpha_i}}{}^{+U}, 1), min(Ƀ_{a_{\alpha_f}}{}^{+U} + (Ƀ_{a_{\alpha_i}}{}^{+U}, 1))], (\left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L}\right),$

$max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U})], [-0,-0],$

$[\min(\max(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, 1), \max(Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-L}, 1), \max(Ƀ_{a_{\alpha_f}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, 1), \max(Ƀ_{a_{\alpha_f}}{}^{-U} + (Ƀ_{a_{\alpha_i}}{}^{-U}, 1)],$

$\subseteq< \left[(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}\right], [0,0], [\max(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L}, 1), \min(Ƀ_{a_{\alpha_f}}{}^{+U} + Ƀ_{a_{\alpha_i}}{}^{+U}, 1)],$

$\left[Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}\right], [-0,-0], [\max(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L}, -1), \max(Ƀ_{a_{\alpha_f}}{}^{-U} + Ƀ_{a_{\alpha_i}}{}^{-U}, -1)] > \cap$

$\left[Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+U}\right], [0,0], [\min(Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+L} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+L}, 1), \min(Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+U} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+U}, 1)],$

$\left[Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U}\right], [-0,-0], [\max(Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-L}, -1), \max(Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-U} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U}, -1)] >$. Thus

$\subseteq \nabla Ƀ_a \cap \nabla Ƀ_{\mathfrak{b}}$.

3) It can prove in similar way in 2.

$\nabla(Ƀ_{a_1} + Ƀ_{\mathfrak{b}}) =$

$= \nabla< \left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+L}, 1\right), \min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{+U}, 1)], [\min\left(Ƀ_{a_{\alpha_i}}{}^{+L} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+L}, 1\right), \min(Ƀ_{a_{\alpha_i}}{}^{+U} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{+U}, 1)], [\min\left(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+L}, 1\right),$

$\min(Ƀ_{a_{\alpha_f}}{}^{+U} + Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{+U})], \left[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} + Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-L}, -1\right), \max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{\mathfrak{b}_{\tilde{\alpha}_t}}{}^{-U}, -1)], \max(Ƀ_{a_{\alpha_i}}{}^{-U} + Ƀ_{\mathfrak{b}_{\alpha_i}}{}^{-U}, 1)], [\max\left(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{\mathfrak{b}_{\alpha_f}}{}^{-L}, 1\right),$

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$\max(Ƀ_{a_{\alpha_f}}{}^{-U} + Ƀ_{b_{\alpha_f}}{}^{-U},1)],>$

$=<\left[\min(Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L},1\right), \min Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U},1)],[0,0],$

$[\min(\min(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{b_{\alpha_f}}{}^{+L},1) + \min\left(Ƀ_{a_{\alpha_i}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L},1\right),1),\min($

$\min(Ƀ_{a_{\alpha_f}}{}^{+U} + Ƀ_{b_{\alpha_f}}{}^{+U},1) + \min\left(Ƀ_{a_{\alpha_i}}{}^{+U} + Ƀ_{b_{\alpha_i}}{}^{+U},1\right),1)],[\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L} +$

$Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L} - 1),\max(Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U} + Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U} - ,1)],[0,0],[\min(\max(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{b_{\alpha_f}}{}^{-L},-1) +$
$\max\left(Ƀ_{a_{\alpha_i}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L},1\right),-1),\min(\min(Ƀ_{a_{\alpha_f}}{}^{-U} + Ƀ_{b_{\alpha_f}}{}^{-U},$

$-1) + \min\left(Ƀ_{a_{\alpha_i}}{}^{-U} + Ƀ_{b_{\alpha_i}}{}^{-U},1\right),-1)],$

$=<\left[Ƀ_{a_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{+U}\right],[0,0], [\min(Ƀ_{a_{\alpha_f}}{}^{+L} + Ƀ_{a_{\alpha_i}}{}^{+L},1),\min(Ƀ_{a_{\alpha_f}}{}^{+U}$

$+ Ƀ_{a_{\alpha_i}}{}^{+U},1)],\left[Ƀ_{a_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{a_{\tilde{\alpha}_t}}{}^{-U}\right],[-0,-0], [\min(Ƀ_{a_{\alpha_f}}{}^{-L} + Ƀ_{a_{\alpha_i}}{}^{-L},-1)$

$<\left[Ƀ_{b_{\tilde{\alpha}_t}}{}^{+L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{+U}\right],[0,0], [\min(Ƀ_{b_{\alpha_f}}{}^{+L} + Ƀ_{b_{\alpha_i}}{}^{+L},1),\min(Ƀ_{b_{\alpha_f}}{}^{+U} + Ƀ_{b_{\alpha_i}}{}^{+U},1)],$

$\left[Ƀ_{b_{\tilde{\alpha}_t}}{}^{-L}, Ƀ_{b_{\tilde{\alpha}_t}}{}^{-U}\right],[-0,-0], [min(Ƀ_{b_{\alpha_f}}{}^{-L} + Ƀ_{b_{\alpha_i}}{}^{-L},-1),min(Ƀ_{b_{\alpha_f}}{}^{-U}$

$+ Ƀ_{b_{\alpha_i}}{}^{-U},-1)] >.$

Therefore$\nabla(Ƀ_a + Ƀ_b) = \nabla Ƀ_a + \nabla Ƀ_b$.

Table (1) Operations on SB-bipolar interval neutrosophic set

$Ƀ_a \cap Ƀ_b$	$\{\min\left(T_{Ƀ_a}(x),T_{Ƀ_b}(x)\right)^{+},\min\left(I_{Ƀ_a}(x),I_{Ƀ_b}(x)\right)^{+},\max\left(F_{Ƀ_a}(x),F_{Ƀ_b}(x)\right)^{+}$ $\max\left(T_{Ƀ_a}(x),T_{Ƀ_b}(x)\right)^{-},\max\left(I_{Ƀ_a}(x),I_{Ƀ_b}(x)\right)^{-},\min\left(F_{Ƀ_a}(x),F_{Ƀ_b}(x)\right)^{-}\}$
$Ƀ_a \cup Ƀ_b$	$\{\max\left(T_{Ƀ_a}(x),T_{Ƀ_b}(x)\right)^{+},\max\left(I_{Ƀ_a}(x),I_{Ƀ_b}(x)\right)^{+},\min\left(F_{Ƀ_a}(x),F_{Ƀ_b}(x)\right)^{+}$ $\min\left(T_{Ƀ_a}(x),T_{Ƀ_b}(x)\right)^{-},\min\left(I_{Ƀ_a}(x),I_{Ƀ_b}(x)\right)^{-},\max\left(F_{Ƀ_a}(x),F_{Ƀ_b}(x)\right)\}^{-}$
$\nabla Ƀ_a$	$\{T_{Ƀ_a}(x)^{+},[0,0],\min((F_{Ƀ_a} + I_{Ƀ_a})^{+},1),T_{Ƀ_a}(x)^{-},[-0,-0],$ $\max((F_{Ƀ_a} + I_{Ƀ_a})^{-},-1)\}$
$\Delta Ƀ_a$	$\{\min(T_{Ƀ_a}(x) + I_{Ƀ_a}(x)),^{+} 1),[0,0],F_{Ƀ_a}(x)^{+},$ $\max\left(T_{Ƀ_a}(x) + I_{Ƀ_a}(x)\right)^{-},-1),[-0,-0],F_{Ƀ_a}(x)^{-}\}$

4. Conclusion

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In this paper, we introduced the concept of SB-bipolar interval neutrosophic set which is an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. The basic operations involving union, complement, intersection, truth-favorite and false-favorite for SB-bipolar interval neutrosophic sets was examined. Moreover, basic properties and theorems and of these operations related to SB-bipolar interval neutrosophic set were presented and mathematically proven.

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